The Yield Spread as a Symmetric Predictor of Output and Inflation

Gikas A. Hardouvelis[*] **and Dimitrios Malliaropulos**[†]

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Abstract :
We present evidence that the predictive ability of the yield spread for short-run inflation is related to its predictive ability for economic activity. In particular, an increase in the slope of the term structure predicts an increase in output growth and a decrease in inflation of equal magnitude. In order to explain this finding, we develop a monetary asset pricing model with sticky goods prices. Sticky prices imply that economic disturbances generate predictable changes in output and inflation, thus allowing for intertemporal substitution effects and changes in the slope of the yield curve. We derive analytic solutions of the covariance between the nominal yield spread and future output growth and inflation and show that a moderate degree of price stickiness and relatively high degree of intertemporal substitution can account for the observed correlations in the U.S. data over the period 1960:Q1 – 2003:Q2.
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[*]University of Piraeus, Economic Office of the Primeminister and CEPR. e-mail: g.hardouvelis@primeminister.gr

[†]University of Piraeus and National Bank of Greece. e-mail: dmaliar@nbg.gr. Address for correspondence: Dimitrios Malliaropulos, Department of Banking and Finance, University of Piraeus, 80 Karaoli & Dimitriou Street, 18534 Piraeus, Greece.

1 Introduction

Following the original independent findings of Chen (1991), Estrella and Hardouvelis (1991), Harvey (1988), and Stock and Watson (1989), a large body of empirical literature has documented that the slope of the yield curve – defined as the difference between long-term and short-term interest rates of Treasury securities – is positively related to future real economic activity.[1] Harvey (1988) has shown that the real yield spread contains information about future consumption growth. Stock and Watson (1989) found that nominal yield spreads have excellent leading indicator properties for economic activity. Estrella and Hardouvelis (1991) documented that the yield spread between 10-year Treasury bonds and 3-month Treasury bills is a good predictor of future growth in output, consumption and investment for horizons up to two years ahead. Estrella and Mishkin (1997) confirmed that the findings of Estrella and Hardouvelis (1991) apply to a number of European countries. Related work by Fama (1990) and Mishkin (1990a,b) shows that the yield spread has some predictive power for future changes in the rate of inflation at horizons of two years and beyond.[2]

In contrast to previous studies, which examine the predictive ability of the yield spread by regressing the *difference* of future inflation at long horizons - two years ahead - from current inflation on the current yield spread, in this paper we examine the relationship of the yield spread with the immediate future *level* of inflation. We regress the future rate of inflation on the current yield spread and discover a new empirical regularity, which previously went unnoticed: *The current level of the yield spread is negatively related to the future level of inflation for horizons between one quarter and one and a half years ahead.*

We provide evidence that the forecasting ability of the yield spread for short-run inflation is related to its forecasting ability for output. An increase in the yield spread leads to an increase in future output and a simultaneous drop in future prices of approximately the same percentage as the percentage increase in real output – see the sample correlations in Figure 1 for an indication of the symmetry. The predictability of inflation and real output

[1] Recent examples are the studies of Estrella and Mishkin (1997), Haubrich and Dombrosky (1996), Plosser and Rouwenhorst (1994), Dueker (1997), Kozicki (1997), Dotsey (1998) and Hamilton and Kim (2000).

[2] See also Mishkin (1991), Jorion and Mishkin (1991), Estrella and Mishkin (1997) and Kozicki (1997), among others..

seem to be mirror reflections of the same economic phenomenon! We confirm this finding by testing the hypothesis that the yield spread is a symmetric predictor of future output and inflation. Indeed, this hypothesis cannot be rejected by the data.

The symmetry in the predictability of output and inflation is further corroborated by the remarkable finding that during periods when the forecasting ability for output deteriorates (especially after the mid-1980s), the forecasting ability of the yield spread for inflation also deteriorates by a similar amount – see the rolling sample regression coefficients of Figure 2.

The symmetric predictability of output and inflation via the yield spread is *a new stylized fact*, which requires an economic explanation.[3] So far the literature has concentrated on providing an economic explanation for the predictability of output. Most authors provide plausible economic stories for the predictability of output. These explanations are not mutually exclusive and it is, therefore, hard to statistically discriminate between them. The new evidence on the symmetric price predictability clearly challenges some of those explanations. Some of this literature is reviewed later in Section 3. Of course, our focus in the paper is not to run a beauty contest among the existing explanations. Our main challenge is to build a general equilibrium model that explains not only output predictability - something the previous literature has so far failed to do - but the symmetric price predictability as well.

We build a parsimonious one-factor general equilibrium model of a monetary economy with sticky prices, which is able to explain the stylized facts as a result of intertemporal smoothing of rational consumers. We make the model as simple as possible and explore how far it can go in explaining the predictive ability of the term structure for output and prices. We derive explicit analytic solutions of the model, which relate the predictive power of the yield spread to two main "deep" structural economic parameters: the degree of price stickiness and the elasticity of intertemporal substitution of the representative consumer.

We subsequently estimate the model parameters and find that moderate price stickiness and relatively strong intertemporal substitution are sufficient conditions for explaining the stylized facts on the symmetric predictability of the yield spread. We also test the model's overidentifying restrictions. These

[3] Our finding is in line with the stylized fact that prices move countercyclically, see King and Watson (1996).

restrictions cannot be rejected by the data.

One key feature of the model is the simplicity of its dynamics. The dynamics are driven entirely by the nature of price stickiness. This distinguishes our model from the class of affine yield models, in which the dynamics of the factors driving the economic variables are exogenous.[4] Because prices are sticky, current economic shocks lead to predictable changes in future prices and output. These expectations, coupled with consumption smoothing and arbitrage, lead to contemporaneous changes in real and nominal interest rates.

A second key feature of the model is that the velocity of money is constant and, thus, productivity and money supply shocks lead to symmetric effects on future output and inflation, a characteristic which is required in order to explain the new empirical evidence of the paper.

A third key feature is the opposite influence of shocks on real and nominal interest rates. Positive productivity shocks increase real but decrease nominal interest rates. Positive money supply shocks decrease real but increase nominal interest rates. As we explain later, this feature is central in explaining the evidence and distinguishes our model from previous models. Indeed, previous unsuccessful attempts at explaining the predictability of output within a general equilibrium model of an endowment economy focused exclusively on real economies and real magnitudes. Yet, the empirical evidence is based on the nominal yield spread, not the real yield spread. Thus the explanation of the evidence requires a monetary model, which can jointly predict output and prices.

To build some intuition on the mechanics of the model and its ability to explain the evidence, consider the effects of a permanent positive productivity shock. This shock increases consumption and output and reduces prices contemporaneously, creating the base of comparisons with future levels of consumption, output and prices. Due to price stickiness, prices have not declined fully to their lower steady state level. In the subsequent periods, they are expected to further decline slowly towards their new steady state. This future further decrease in prices is expected to lead to a symmetric -

[4]One recent example of this type of models is Ang et al. (2003). In this paper, bond yields are derived as affine functions of a state vector, consisting of GDP growth, the yield spread and the short-term interest rate. The dynamics of the factors are governed by a Vector Autoregressive model. The authors find that imposing no-arbitrage restrictions increases the forecasting power of the yield curve for future GDP, compared to unrestricted OLS.

due to the constant velocity of money - further increase in consumption and output in every future period, albeit at smaller and smaller magnitudes as time goes on, which reflect the ever smaller declines in prices. Real interest rates increase because the expected increase in future consumption relative to today's consumption decreases the marginal utility of future consumption relative to today's marginal utility. This leads rational agents to borrow and consume more today in order to smooth their consumption, hence, pushing upward the real rate of interest. However, if the elasticity of intertemporal substitution of consumption is high (larger than unity), the increase in real interest rates, which is required in order to bring about this consumption smoothing, is not very large and is overwhelmed by the drop in expected inflation. Hence, nominal interest rates decline. All nominal rates decline across the full maturity spectrum, but short rates decline a lot more than long rates. This is because, as was explained above, as time passes, the expected change in prices and output washes out gradually. The largest impact occurs early on and influences current short rates a lot more than expected future short rates. Since long rates are weighted averages of current and expected future short rates, the impact on them will be smaller than the impact on short rates. It follows that the yield spread increases. So we end up with a current increase in the yield spread, a decrease in expected future prices and a symmetric increase in expected future output.

The remainder of the paper is as follows: Section 2 presents the stylized facts about the predictive ability of the yield spread for output and inflation. Section 3 contains a brief review of the theoretical literature. Section 4 presents the asset-pricing model and derives analytic solutions of the covariance between the yield spread and future output growth and inflation. Section 5 presents the empirical estimates of the model. Section 6 concludes and discusses possible extensions.

2 Old and New Stylized Facts

2.1 Data

The empirical analysis is based on quarterly data for the United States from 1960:Q1 to 2003:Q2. Data are from the Federal Reserve Bank of St. Louis (FRED) database. As a measure of economic activity, we use seasonally adjusted data on real, chain-weighted Gross Domestic Product (GDP), expressed in 1996 prices. Prices are measured by the seasonally adjusted Consumer Price Index (CPI). Long-term interest rates are annualized yields to maturity of 3-year, 5-year and 10-year Treasury Bonds. The yield spread is computed as the difference between long-term interest rates and the 3-month Treasury bill rate. All data are quarterly averages. Table 1 reports summary statistics and correlations of our data. The yield spread is positively correlated with one-year ahead GDP growth, with correlations ranging between 0.40 and 0.44, and negatively correlated with one-year ahead inflation, with correlations between -0.32 to -0.36.

2.2 Old Evidence: Output Predictability

Formal evidence of the predictive ability of the yield spread for future GDP growth is presented in Table 2. The table reports estimates of the typical OLS regression used by most researchers to measure the predictive ability of the yield spread for future output:

$$(y_{t+k} - y_t)100(\frac{4}{k}) = a_0 + a_1 s_t + u_{y,t} \tag{1}$$

where y_t is log real GDP, $(y_{t+k} - y_t)100(\frac{4}{k})$ measures the annualized growth rate of real GDP from quarter t to quarter $t+k$ and s_t is the yield spread, measured as the difference between long-term interest rates and short-term interest rates.

Panels A, B and C of the table report estimated slope coefficients and adjusted $R^{2\prime}s$ of regression (1) with the 3-year, 5-year and 10-year yield spread, respectively. The estimates are qualitatively similar to those obtained by a number of previous researchers, confirming that the yield spread has predictive power for future GDP growth up to three years ahead. The adjusted $R^{2\prime}s$ peak at k between four and eight quarters, suggesting that the predictive ability of the yield spread is highest for forecasting horizons between one and two years. Economically, an increase in the 10-year yield spread by 100 basis points predicts an increase in output growth by about 0.7 percentage points in two years' time. This elasticity is higher for shorter maturities.

Panel D of the table reports estimation results of the regression of one year ahead GDP growth on the 10-year yield spread for various sub-periods. The estimates suggest that the ability of the yield spread to predict one year ahead GDP growth broke down during the 1990s, confirming the results of

Haubrich and Dombrosky (1996) and Dotsey (1998). However, this predictive ability may be coming back after the end of the prolonged expansion of the 1990's.[5]

2.3 New Evidence: Short-Run Inflation Predictability

Evidence on the predictive power of the yield spread for inflation is not as strong as it is for real activity. Fama (1990) and Mishkin (1990a,b) examined the ability of the yield curve to predict future changes in inflation. Their approach is based on the Fisher decomposition: if nominal interest rates are positively related to expected inflation, then the yield spread between a bond with k periods to maturity and the one-period interest rate should contain information about the change in expected inflation between time $t+1$ and time $t+k$. In particular, an increase in the slope of the term structure should predict an increase in expected inflation. The empirical results for the US suggest that the yield spread has some predictive ability at horizons of two years and beyond. Mishkin (1991), Jorion and Mishkin (1991), Estrella and Mishkin (1997) and Kozicki (1997), among others, extended the analysis to a number of countries other than the US and obtained similar results.

In contrast to these studies, which test the predictive ability of the yield spread by regressing the future *change* in inflation on the current yield spread, we regress the future *level* of inflation on the current yield spread and discover a new empirical regularity: an increase in the current level of the yield spread is negatively related to the future level of inflation for horizons between one quarter and one and a half years ahead.

Table 3 reports estimation results of the relationship between the yield spread and future inflation. The table reports estimates of the OLS regression:[6]

$$(p_{t+k} - p_t)100(\frac{4}{k}) = a_0 + a_1 s_t + u_{p,t} \tag{2}$$

[5] The yield spread did in fact a good job in predicting the 2001 recession. The estimated slope coefficient in (1) over the period 2000:Q1-2003:Q2 is 0.70 with a standard error of 0.15 and an $\overline{R}^2$ of 0.57. Of course, the number of observations is still too small to make any reliable inference.

[6] It is clear that this regression makes sense only if inflation and the yield spread are stationary variables. Later, in Section 5.1, we present evidence that inflation is stationary after adjusting for a structural break in the mean and the deterministic trend in the early 1980's.

where p_t is the log of the Consumer Price Index.

Panels A, B and C of the table report estimated slope coefficients and adjusted $R^{2\prime}s$ of regression (2) with the 3-year, 5-year and 10-year yield spread, respectively. The estimates suggest that the yield spread has predictive power for up to one and a half year ahead inflation. In particular, an increase of the 10-year yield spread by 100 basis points predicts a decrease in consumer price inflation by slightly more than one percent in one quarter and by about 0.7 per cent in one and a half years. In contrast to the predictive regressions of GDP growth, the adjusted $R^{2\prime}s$ peak at short horizons between one and four quarters, suggesting that the yield spread predicts short-term inflation.[7] The estimates of the predictive regression over various sub-periods presented in Panel D of the table suggest that, similar to the results for output growth, the predictive ability of the yield spread for future inflation broke down during the 1990s.

2.4 Is the Predictability of Output and Inflation Symmetric?

Comparing the estimates of the slope coefficients of regressions (1) and (2), one can observe a striking symmetry: an increase in the yield spread predicts opposite changes in real GDP and consumer prices of almost the same magnitude, i.e. $b_1 = -a_1$. This hypothesis can be easily tested, since, adding regressions (1) and (2), we obtain the regression:

$$(y_{t+k} + p_{t+k} - y_t - p_t)100(\frac{4}{k}) = c_0 + c_1 s_t + u_{yp,t} \tag{3}$$

where $c_0 = a_0 + b_0$ and $c_1 = a_1 + b_1$. The null hypothesis that the yield spread predicts opposite changes in real GDP and prices, i.e. $b_1 = -a_1$, can be tested using regression (3) as $H_0 : c_1 = 0$ by means of a standard $t-$test.

The second row of Table 3 reports $t-$tests of the symmetry hypothesis $H_0 : c_1 = 0$. Independently of the yield spread used and the forecast horizon,

[7] We have also tested the ability of the yield spread to predict future GDP price inflation using regression (2). The results are similar to those of the predictive regressions for consumer price inflation, reported in Table 3. In order to control for autocorrelation in the inflation series, we have also estimated regression (2) with the lagged dependent variable, $(p_t - p_{t-k})100(\frac{4}{k})$, as an additional regressor (not reported). Although the lagged inflation term is significant in all forecasting horizons, the predictive ability of the yield spread is not affected.

the t-statistics are far from any conventional significance level, suggesting that the yield spread is indeed a symmetric predictor of output and prices. Looking at the sub-period results, reported in Panel D of the table, the symmetry hypothesis cannot be rejected in any sub-sample.

Summing up, our preliminary data analysis suggests that there are two stylized facts about the predictive power of the yield spread which require an economic explanation: the positive association between the yield spread and future economic activity, and the symmetric negative association between the yield spread and future inflation.

3 Existing Literature: Can a Consumption Based Asset Pricing Model Explain the Stylized Facts?

A natural candidate for explaining the joint behavior of output, prices and the term structure of interest rates is a general equilibrium asset pricing model of a monetary economy. A successful model should explain the property of the yield spread as a symmetric predictor of output and prices in terms of deep economic parameters and structural shocks. To our best knowledge, however, no such model has been proposed thus far, perhaps because the new stylized fact on the symmetric predictability of the spread had escaped the attention of earlier researchers.

Early attempts to explain the correlation of the yield spread and subsequent output or consumption growth essentially provided heuristic stories of the correlation. Estrella and Hardouvelis (1991), for example, interpret the positive association between the yield spread and future output growth as arising from market expectations of future shifts in investment opportunities and output (an expected future shift in the IS curve that would affect future output and future short rates, hence the current long rate). They claim the association is not due to the current behavior of the central bank (a current shift in the LM curve, which affects short-term rates and future economic activity), claiming they have controlled for the central bank's behavior in their regression analysis. Later on, Estrella (1998, 2003) built models in which the behavior of the central bank is important. Estrella examines a macroeconomic model consisting of an IS curve, a Phillips curve and a monetary policy reaction function and shows that the predictive power of the yield spread depends on the preferences of the central bank and, in particular, on the importance of inflation targeting relative to the importance of output stabilization in the monetary policy rule.

In this section we review the literature, which is based on models of the real economy because later, in Section 4, we expand on those models by introducing money. Harvey (1988) sketches the first such model, using the consumption-based CAPM. Others have subsequently followed his lead (Hu (1993), Den Haan (1995), De Lint and Stolin (2003) and Estrella, Rodrigues and Schich (2003)). The basic capital asset pricing model of Lucas (1978) predicts that in an endowment economy, asset prices are linked to the marginal utility of the representative agent, which depends on consumption.[8] With time-separable power utility, the first-order condition of a representative consumer establishes a positive correlation between the $\tau-$period bond yield at time t and expected consumption growth between period t and period $t+\tau$. If agents expect a decline in their consumption τ periods from now, they will sacrifice part of today's consumption in order to buy a bond which pays off in τ periods from today. The demand for the bond will bid up its price and lower its real yield. This positive association is depicted as follows:

$$\rho(\tau)_t = \alpha_\tau + \frac{\gamma}{\tau} E_t(c_{t+\tau} - c_t) \tag{4}$$

where $\rho(\tau)_t$ is the real yield to maturity of a $\tau-$period bond, $\frac{1}{\tau}E_t(c_{t+\tau}-c_t)$ is the average expected growth rate of consumption between period t and period $t+\tau$, α_τ is a constant and γ is the reciprocal of the elasticity of intertemporal substitution between present and future consumption.

As De Lint and Stolin (2003) explain in detail, the positive relation between the level of real interest rates and expected future consumption growth does not imply an analogous positive relation between the yield spread and future consumption growth. To see this, rewrite equation (4) for the case of $\tau = 1$, and subsequently subtract the result from (4):

$$\rho(\tau)_t - \rho(1)_t = \alpha + \gamma \left[\frac{1}{\tau} E_t(c_{t+\tau} - c_t) - E_t(c_{t+1} - c_t) \right] \tag{5}$$

Observe that the left-hand-side of equation (5) is, indeed, the real yield spread or the slope of the real term structure. However, the item in brackets in the right-hand-side of the equation is the expected *difference* between average growth in consumption over τ periods and the one-period growth. It

[8]This is also the prediction of Real Business Cycle Models, see, for example Kydland and Prescott (1988), among many others.

is *not the level* of average expected consumption growth. Unlike (4), in (5) the relation between the left-hand-side and right-hand-side variables is *negative*, not positive. Suppose, for example, that consumption *growth* follows an autoregressive process of order one, with an autoregressive parameter ϕ, $0 < \phi < 1$. Then, equation (5) becomes:

$$\rho(\tau)_t - \rho(1)_t = \alpha - \gamma \left[1 - \frac{1}{\tau}(1 + \phi + \phi^2 + \ldots + \phi^{\tau-1}) \right] E_t(c_{t+1} - c_t) \quad (6)$$

The slope coefficient in the above relation is always negative. This is because the positive impact of a consumption shock on expected consumption growth and, hence, the negative impact on the expected marginal rate of substitution (MRS) is the highest during the period when the shock occurs and subsequently declines monotonically. The decrease in the MRS, induces consumers to borrow, driving up the real rates of interest across the full maturity spectrum. However, the short-term real interest rate increases by more that the long-term rate does, leading to a decline in the real yield spread. Hence, the yield spread is negatively associated with the expected future growth in consumption. In the above formulation, the only way the yield spread could ever be positively related to the expected future growth in consumption is if the autoregressive parameter were larger than unity. However, $\phi > 1$ would imply an explosive process for the growth rate of consumption, which is contradicted by the evidence.[9]

Clearly, in order to accommodate the empirical finding that the yield spread is positively related to the expected future growth in consumption and output within the confines of a real economy, one would have to enrich the model in such a way that the influence of shocks on the marginal rates of substitution between period t and successive periods $t + \tau$ increases as the

[9]The response of the level of the real rate of interest to a consumption shock is a function of the assumed persistence of the shock. De Lint and Stolin (2003) assume that the level of log consumption is an autoregressive process as opposed to the growth in consumption. In this case, the prediction of the change in the level of the real interest rate changes direction, but the prediction on the relationship between the spread and the expected future growth in consumption remains negative. In the De Lint and Stolin model, following a positive shock on consumption, today's consumption is higher that all future levels of consumption. Hence, the MRS increases for every forecasting horizon τ, producing a drop in the real rates of interest across the full maturity spectrum. However, the increase in the MRS is smaller the longer the forecasting horizon, hence short rates decrease more than long rates, leading to an increase in the spread. Thus, the higher spread is negatively associated with the lower expected future growth in consumption.

forecasting horizon τ lengthens. This is not easy to do in a manner that is robust and intuitive.

Following Den Haan (1995), De Lint and Stolin (2003) propose expanding the model to include production. In a production economy, the decision to forego consumption today can lead to additional consumption in the future via a second channel, besides lending in the bond market. This channel is via investing the proceeds of the sale of the consumption good and increasing the capital stock, thus ending up in the future with the marginal product of the additional capital as well as the depreciated amount of the extra capital. In the model, the influence of a shock gets distributed over time via the investment channel, altering the monotonic nature of the marginal rates of substitution that we encountered in the endowment economy. De Lint and Stolin provide simulation results, which show a positive association between the spread and future consumption growth only for forecasting horizons longer than 10 years! For shorter forecasting horizons the association is negative. Yet, the empirical evidence on the positive association is primarily manifested in the short-run, mainly over forecasting horizons between one and two years ahead (see Figure 1).

Estrella, Rodrigues and Schich (2003) propose expanding the model to include habit formation, in a manner similar to Campbell and Cochrane (1999). In this model, utility depends on the difference of consumption from habit and habit is a complex non-linear function of current and past consumption shocks. In the model, the marginal rates of substitution are no longer simple functions of the expected growth in consumption, but depend also on the evolution of the consumption surplus ratio, defined as $S_t = (C_t - X_t)/C_t$, where X_t represents the level of habit. The log of the consumption surplus ratio, s_t, is assumed to follow an AR(1) process with autoregression parameter ϕ_s. Campbell and Cochrane assume that the growth in consumption follows a random walk with a drift parameter g. They are subsequently able to derive the following simple expression for the real rate of interest with maturity τ periods:

$$\rho(\tau)_t = \beta_\tau + \gamma g - \frac{\gamma}{\tau}(1 - \phi_s^\tau)(s_t - \overline{s}) \tag{7}$$

where $\overline{s}$, is the steady state value of s_t. When the consumption surplus ratio is above its steady state value, the marginal utility of consumption is low. Also, the surplus consumption ratio is expected to revert back to its mean, so marginal utility is expected to increase in the future, driving up the

MRS, which leads to a drop in the real rate of interest. Rewriting equation (7) for the case of τ= 1 and subtracting the result from (7), we get:

$$\rho(\tau)_t - \rho(1)_t = \beta + \gamma \left[1 - \phi_s - \frac{1}{\tau}(1 - \phi_s^\tau)\right](s_t - \overline{s}) \tag{8}$$

In equation (8), the yield spread is a positive function of the surplus consumption ratio. This is because following a shock, the real short-term rate declines by more than the long rate does, due to the fact that the MRS is affected more strongly early on before the effect of the shock washes out monotonically. Estrella et.al. subsequently argue that "the surplus consumption ratio tends to drop on bad news about the future. Consequently, more negative yield slopes will tend to precede anticipated deterioration in households' future income flows, and more positive yield slopes will precede good economic environments." This argument, however, is very heuristic and is not supported by the model they present. In the model, log consumption follows a random walk, so the yield spread cannot predict future changes in consumption and output. In fact, it is mean-reversion in the level of habit – and, hence, the MRS – and not mean-reversion in consumption itself which generates changes in the slope of the term structure. Reworking the model, by imposing some autocorrelation structure on consumption growth that would make consumption predictable is a must, before any claims can be made.

Summing up, previous models, which relate the predictive ability of the yield spread to intertemporal consumption smoothing, have focused on the relationship between the real yield spread and future output in a simple endowment economy. These models have not been successful in explaining the evidence. There is a clear need to introduce an economic mechanism that would generate endogenous predictions for both output and price changes and would explain their association with the nominal yield spread. This we do next.

4 A Monetary Asset Pricing Model with Price Rigidities

Motivated by the evidence presented earlier in Section 2, this section sets up a dynamic general equilibrium model, which relates the two stylized facts of the predictive ability of the yield spread: The predictability of output and the symmetric predictability of inflation. In an attempt to keep the model structure as simple as possible, we restrict the analysis to a one-factor equilibrium model of the term structure.

Our theoretical framework is a modification of Rotemberg (1982, 1996). In Rotemberg's model, prices are sticky in the short term due to the existence of costs of price adjustment.[10] We modify the model by using a power utility function and by adding a bond market, in which households can borrow or lend their proceeds for $1, ..., N$ periods. The existence of price rigidities implies that shocks to output and money supply lead to forecastable changes in future price and output growth. As a result, consumers adjust their savings in an attempt to smooth consumption over time, implying a correlation between the current yield spread and future economic activity and inflation.

4.1 The Model

The economy is populated by identical, infinitely-lived households. Each household produces a type of intermediate good which is an imperfect substitute for the other goods and sells it under conditions of monopolistic competition. Prices of intermediate goods adjust with a lag to changes in demand and costs of production due to the existence of a cost of adjusting prices. Firms purchase intermediate goods from households and use them to produce a single consumption good with a constant returns to scale technology. Households can buy or sell nominally risk-free $\tau-$ period discount bonds which promise to pay one dollar in all states of the world at time $t+\tau$, $\tau = 1, ..., N$. Consumption goods must be paid for with money, i.e. households are subject to a Cash-In-Advance constraint. Money is a non-interest bearing security. Each period, the central bank makes a lump-sum money transfer to households.

Final goods

Let Y_t be the output of the final good. It is produced using a continuum of

[10] There are two main specifications of nominal price rigidity, the partial adjustment model (Rotemberg (1982), Calvo (1983)) and staggered price setting (Taylor (1980)). In the partial adjustment model of Rotemberg (1982), firms face quadratic costs of changing prices whereas in Calvo (1983) firms adjust prices according to some constant hazard rate. Under staggered price setting, all firms adjust prices after some fixed period of price rigidity. Rotemberg (1996) tests the main implications of price stickiness in a VAR framework and finds that a sticky price model can be easily reconciled with the most important features of output, price, and hours comovements over the business cycle.

intermediate goods as inputs indexed by $i \in [0,1]$. The production function of the final good is given by:

$$Y_t = \left[\int_0^1 Y_t(i)^q di \right]^{\frac{1}{q}}, \qquad 0 < q \leq 1 \tag{9}$$

where $Y_t(i)$ is the input of intermediate good i and $1/(1-q)$ is the elasticity of substitution between goods. Final goods are produced under conditions of perfect competition. Firms take prices as given and choose $Y_t(i)$ in order to maximize profits, given by: $P_t Y_t - \int_0^1 P_t(i) Y_t(i) di$, where P_t is the price of the final good and $P_t(i)$ is the price of intermediate good i. The resulting demand functions for intermediate goods have the form:

$$Y_{i,t} = Y_t \left(\frac{P_t}{P_{i,t}} \right)^{\frac{1}{1-q}} \tag{10}$$

Intermediate goods

Households produce intermediate goods using labor, $L_{i,t}$, as the only input, according to the production function

$$Y_{i,t} = L_{i,t} \cdot X_t \tag{11}$$

where X_t is a productivity shock. There is monopolistic competition in the market for intermediate goods. Households face the demand curve given by equation (10) and set the price $P_{i,t}$ in order to maximize their utility function.

The utility function of the representative household depends on consumption of the final good, leisure (which we model directly as disutility of work) and negatively on the cost of adjusting prices.

Utility of household i is given by:

$$\begin{aligned} U_{i,t} &= E_t \sum_{k=0}^{\infty} \beta^k \{ \frac{1}{1-\gamma} C_{i,t+k}^{1-\gamma} - \psi X_{t+k}^{1-\gamma} \frac{1}{1-n} L_{i,t+k}^{1-n} \\ &\quad - \frac{c}{2} X_{t+k}^{1-\gamma} [\ln P_{i,t+k} - \ln P_{i,t+k-1}]^2 \} \end{aligned} \tag{12}$$

where E_t is the conditional expectations operator given information up to time t, $\beta \in (0,1)$ is a discount factor, γ is the coefficient of relative risk aversion (which, with power utility is equal to the reciprocal of the elasticity of intertemporal substitution), n is the elasticity of labor supply w.r.t. real wages and ψ and c are positive constants with c depending positively on the cost of price adjustments. As in Rotemberg (1996), we add a multiplicative productivity shock in the two disutility terms in order to ensure that technological progress does not lead to a secular decrease in labor input.[11]

The timing of events is similar to Svensson's (1985) cash-in-advance model: the household enters period t with predetermined money holdings, $M_{i,t}$, and a predetermined portfolio of bonds, $B_{1,t-1}$, $B_{2,t-2}$, ..., $B_{N,t-N}$, acquired during earlier periods. We will allow for the existence of N different bonds (loans) maturing in $1, 2, 3, ...N$ periods. Each of these bonds promises to pay one sure nominal dollar in all states of the world at maturity.

During period t, the household chooses the size of each of the N different loans: $B_{1,t}$, $B_{2,t}$, ..., $B_{N,t}$. The household also receives income from the loans offered during earlier periods. The gross nominal interest received at time t is $\sum_{\tau=1}^{N} R_{\tau,t-\tau}B_{\tau,t-\tau}$, where $B_{\tau,t-\tau}$ is the $\tau-$period bond purchased at time $t-\tau$ and $R_{\tau,t-\tau}$ is the gross nominal interest rate (not annualized) of this bond. This income is known at time t.

The goods market opens first at the beginning of period t – which we subsequently denote by $t-1$ – and the household has the opportunity to purchase the consumption good with money at a price P_{t-1}. His purchases, $C_{i,t-1}$, must obey the cash-in-advance constraint:

$$C_{i,t-1} = M_{i,t-1}/P_{t-1} \tag{13}$$

Since money has no other use than facilitating transactions of goods, the cash-in-advance constraint is binding, so that real money balances acquired during the previous periods determine consumption.

After the goods market is closed at the end of period t – which we denote in the following by t –, the household receives the lump-sum money transfer, $T_{i,t}$. Finally, at the end of period t, money and bond markets open. The household then faces the following intertemporal budget constraint:

$$M_{i,t} = P_{i,t-1}Y_{i,t-1} - P_{t-1}C_{i,t-1} + M_{i,t-1} + T_{i,t} + \sum_{\tau=1}^{N} R_{\tau,t-\tau}B_{\tau,t-\tau} - \sum_{\tau=1}^{N} B_{\tau,t} \tag{14}$$

Hence, the household allocates its income net of consumption, $P_{i,t-1}Y_{i,t-1} + T_{i,t} + \sum_{\tau=1}^{N} R_{\tau,t-\tau}B_{\tau,t-\tau} - P_{t-1}C_{i,t-1}$, between new money holdings, $M_{i,t} -$

[11] See Rotemberg (1996) p. 509 for a discussion.

$M_{i,t-1}$, and new bonds, $\sum_{\tau=1}^{N} B_{\tau,t}$. The income of the household consists of the proceeds from selling its product, $P_{j,t-1}Y_{j,t-1}$, plus the lump-sum money transfer, $T_{i,t}$, plus interest income from the loans it offered during earlier periods, $\sum_{\tau=1}^{N} R_{\tau,t-\tau}B_{\tau,t-\tau}$.

Maximizing (12) w.r.t. $B_{\tau,t}$ and $P_{i,t}$ subject to the constraints (13) and (14) and loglinearizing leads to the following optimality conditions, evaluated at the symmetric equilibrium where $P_{i,t} = P_t$ and $Y_{i,t} = Y_t$:

$$r(\tau)_t = -\log(\beta) + \frac{1}{\tau}\left[\gamma E_t(c_{t+\tau} - c_t) + E_t(p_{t+\tau} - p_t)\right] + \theta \tag{15}$$

$$p_t = \alpha p_{t-1} + (1-\alpha)(1-\delta)E_t \sum_{k=0}^{\infty} \delta^k (m_{t+k} - x_{t+k}) \tag{16}$$

where $r_t(\tau) = \log(R_{\tau,t})/\tau$ is the continuously compounded, annualized yield to maturity at time t on a nominal discount bond with term τ, $c_{t+\tau} - c_t \equiv \log(C_{t+\tau}/C_t)$, $p_{t+\tau} - p_t \equiv \log(P_{t+\tau}/P_t)$, θ is a constant term premium, $\alpha \in (0,1)$ is the degree of price stickiness and $\delta \in (0,1)$ is a constant.[12]

Equation (15) is the well-known condition of Consumption-CAPM. It says that the yield to maturity of a τ-period nominal discount bond at time t is determined by expected average consumption growth and inflation between time t and time $t + \tau$.

Equation (16) says that prices are a linear combination of lagged prices and long-run equilibrium prices, which are given as the discounted value of expected excess money supply over productivity. An expected increase in money supply increases current prices because it increases demand for the final product. An expected increase in productivity decreases current prices because it decreases production costs per unit output. Due to the existence of costs of price adjustment, there is a lagged adjustment of prices towards their long-run equilibrium. The speed of this adjustment depends negatively on the degree of price stickiness, α.

[12]See Appendix A for a derivation. Note that, in general, the parameter θ is time-varying, $\theta_t \equiv -\frac{1}{2\tau}\left[\gamma^2 var_t(c_{t+\tau} - c_t) + var_t(p_{t+\tau} - p_t) + 2\gamma cov_t(c_{t+\tau} - c_t,\ p_{t+\tau} - p_t)\right]$. However, when we loglinearize the optimality condition, we make the assumption that the joint conditional distribution of consumption and prices is i.i.d. lognormal. Hence, the conditional variance and covariance terms are constant and, as a result, the term premium is equal to a constant $\theta = -\frac{1}{2}\left[\gamma^2\sigma_c^2 + \sigma_p^2 + 2\gamma\sigma_{cp}\right]$, where σ_c and σ_p are the variance of c and p and σ_{cp} is their covariance.

In order to derive a price equation in terms of observables, we specify the stochastic processes driving money supply and productivity:

$$m_t = \mu_m + m_{t-1} + \varepsilon_{m,t} \tag{17}$$

$$x_t = \mu_x + x_{t-1} + \varepsilon_{x,t} \tag{18}$$

Money supply and productivity follow random walks with drift factors μ_m, μ_x and independent innovation processes $\varepsilon_{m,t}$ and $\varepsilon_{x,t}$, respectively. Taking expectations of equations (17) and (18) conditional on information up to time t gives: $E_t(m_{t+k}) = m_t + k\mu_m$, $E_t(x_{t+k}) = x_t + k\mu_x$ for all $k = 0, \ldots, \infty$. Substituting in equation (16), we obtain:

$$p_t = \alpha p_{t-1} + (1-\alpha)(m_t - x_t) + \frac{\delta(1-a)\mu}{1-\delta} \tag{19}$$

where $\mu \equiv \mu_m - \mu_x$. Taking first differences of equation (19) gives the rate of inflation as a function of contemporaneous and past innovations to money supply and productivity:

$$\Delta p_t = \frac{(1-\alpha)}{(1-\alpha L)}(\Delta m_t - \Delta x_t) = \mu + \psi(L)(\varepsilon_{m,t} - \varepsilon_{x,t}) \tag{20}$$

where $\psi(L) = (1-\alpha)/(1-\alpha L)$ is an infinite-order polynomial in the lag operator L, such that $\psi(L)z_t = (1-\alpha)(z_t + \alpha z_{t-1} + \alpha^2 z_{t-2} + \cdots + \alpha^p z_{t-p} + \cdots)$.[13] Note that $\psi(1) = 1$, meaning that a one-off monetary shock leads to a proportional long-run increase in the price level, whereas a one-off productivity shock leads to a proportional decrease in the price level. The conditional expectation of the long-run rate of inflation is given by $E_t \Delta p_{t+\infty} = \mu + \varepsilon_{m,t} - \varepsilon_{x,t}$ (long-run quantity theory).

The relationship between output, money and prices is given by the Cash-In-Advance constaint, equation (13), together with the condition that in equilibrium consumption is equal to output, i.e., in logs: $y_t = m_t - p_t$. Substituting (19) in this equation for p_t and taking first differences, we obtain:

$$\Delta y_t = \mu_x + (1 - \psi(L))\varepsilon_{m,t} + \psi(L)\varepsilon_{x,t} \tag{21}$$

According to equation (21), real output growth is a function of current and past monetary and productivity shocks. Since $\psi(1) = 1$, the monetary shock, $\varepsilon_{m,t}$, represents the transitory component, whereas the productivity shock, $\varepsilon_{x,t}$, represents the permanent component of output growth.

[13] The lag operator L is defined as: $L^i z_t \equiv z_{t-i}$.

4.2 Why does the Yield Spread Predict Future Economic Activity and Inflation?

In order to derive the term structure of interest rates as a function of unexpected changes in money supply and productivity, we first compute the conditional expectation of the continuously compounded output growth rate and inflation.

From equations (21) and (20) we obtain for the conditional expectation of the growth rate of output (consumption) and prices from period $t+h-1$ to period $t+h$ for $h \geq 1$:

$$E_t(\Delta y_{t+h}) = -E_t(\Delta p_{t+h}) = \alpha^h \psi(L)\varepsilon_t \tag{22}$$

where, for convenience, we have skipped the constants and re-defined the innovation process as $\varepsilon_t \equiv \varepsilon_{x,t} - \varepsilon_{m,t}$.

Hence, the continuously compounded, annualized rate of output growth between time t and time $t+h$, given information up to time t, is:

$$\frac{1}{h}E_t(y_{t+h} - y_t) = -\frac{1}{h}E_t(p_{t+h} - p_t) = \kappa(h)\psi(L)\varepsilon_t \tag{23}$$

where $\kappa(h) = \frac{\alpha(1-\alpha^h)}{h(1-\alpha)}$.

Next, setting $h = \tau$ in (23) and substituting the resulting equation in (15), we obtain for the time t yield to maturity of a $\tau-$period nominal discount bond:

$$r(\tau)_t = -\log(\beta) + \kappa(\tau)(\gamma - 1)\psi(L)\varepsilon_t + \theta \tag{24}$$

where $\kappa(\tau) = \frac{\alpha(1-\alpha^\tau)}{\tau(1-\alpha)}$.

Using equation (24) and noting that $\kappa(1) = \alpha$, the yield spread, defined as $s_{\tau,t} = r(\tau)_t - r(1)_t$, can be written as:

$$s_{\tau,t} = (\kappa(\tau) - \alpha)(\gamma - 1)\psi(L)\varepsilon_t \tag{25}$$

Equation (25) demonstrates that the effect of productivity shocks and monetary shocks on $s_{\tau,t}$ depends on the degree of price stickiness, α, the elasticity of intertemporal substitution, γ^{-1}, and term to maturity, τ. In order to understand how economic shocks affect the yield spread, first note that $\kappa(\tau) - \alpha < 0$ for $0 < \alpha < 1$, $\tau > 1$, implying that long-term interest rates react less strongly than one-period interest rates to a productivity shock or a monetary shock. This occurs because most of the change in expected inflation and output takes place in the first periods following the shock, implying that the average expected one-period interest rate over the next τ periods changes less than the current one-period interest rate.

A positive productivity shock increases output and decreases consumer prices permanently. Hence, its impact on nominal interest rates depends on the relative importance of these two effects. While the real short-term interest rate increases by $\gamma\alpha$, one period ahead expected inflation decreases by α. The combined effect of a productivity shock on the nominal short-term interest rate is $\alpha(\gamma - 1)$, whereas its effect on the nominal long-term interest rate is $\kappa(\tau)(\gamma-1)$. As a result, the effect on the yield spread is $(\kappa(\tau)-\alpha)(\gamma-1)$. Since $(\kappa(\tau)-\alpha) < 0$, the effect of a shock depends on the elasticity of intertemporal substitution, γ^{-1}. If agents have logarithmic preferences ($\gamma = 1$), the increase in the real rate will exactly offset the decrease in expected inflation, leaving nominal interest rates and the nominal yield spread unchanged. In this case, changes in expected inflation exactly offset changes in the real yield spread. If $0 < \gamma < 1$ (i.e. the elasticity of intertemporal substitution, $\gamma^{-1} > 1$), real interest rates increase less than the reduction in expected inflation, producing lower nominal interest rates. The nominal yield spread widens, although the real yield spread shrinks.

A positive monetary shock increases expected inflation and decreases real interest rates. If $0 < \gamma < 1$, short-term real rates will decrease less than the rise in expected inflation, producing higher nominal interest rates. The nominal yield spread shrinks, while the real yield spread widens.

In order to assess the economic determinants of the predictive power of the yield spread for future output and inflation, we compute the conditional covariance between the time t yield spread, given by equation (25) and the $h-$period ahead continuously compounded annualized output growth, $\frac{1}{h}E_t \sum_{i=1}^{h} \Delta y_{t+i} \equiv \frac{1}{h}E_t(y_{t+h} - y_t)$, and inflation, $\frac{1}{h}E_t \sum_{i=1}^{h} \Delta p_{t+i} \equiv \frac{1}{h}E_t(p_{t+h} - p_t)$, given by equation (23). Noting that the innovations are i.i.d. with constant variance σ_ε^2, the conditional covariance between the time t yield spread and the $h-$period ahead continuously compounded annualized output growth is:

$$Cov_t(s_{\tau,t},\ \frac{1}{h}(y_{t+h} - y_t)) \quad = \quad -Cov_t(s_{\tau,t},\ \frac{1}{h}(p_{t+h} - p_t))$$

$$= \kappa(h)(\kappa(\tau) - \alpha)(\gamma - 1)\frac{(1-\alpha)^2}{1-\alpha^2}\sigma_\varepsilon^2 \qquad (26)$$

Figure 3 displays the covariance between the yield spread and future output growth for various values of α, ranging from zero to one. In the figure we set $\tau = 40$ and $h = 4$, in line with empirical evidence that the yield spread between 10-year (40 quarters) bonds and 3-month bills is a good predictor of 4 quarter ahead GDP growth. Furthermore, we set $\sigma_\varepsilon^2 = 3.6$, the sample variance of the difference in innovations of quarterly changes in GDP and M3 money sypply.[14] Finally, we set $\gamma = 0.6$, in line with our estimates reported below.[15]

There are several results worth noticing from equation (26) and Figure 3. First, the covariance is negative for values of γ exceeding unity, or, equivalently, for values of the elasticity of intertemporal substitution below unity, since in this case agents require a strong change in real interest rates in order to substitute present for future consumption. Hence, given a shock at time t which increases future output and decreases future prices, the real yield spread declines more than future inflation, leading to a narrowing of the nominal yield spread.

Second, the covariance decreases both in maturity, τ, and the forecast horizon, h, in line with the empirical evidence reported in Tables 2 and 3 that the slope coefficients of the output and price forecast regressions decline symmetrically as the term of the yield spread increases and the forecast horizon lengthens.

Third, the covariance between the yield spread and output growth is highest for intermediate values of the degree of price stickiness α. In order to understand this result, note that for low levels of α prices adjust faster to shocks, leading to a smaller change in both future output and future prices. As a result, the slope of the yield curve responds less to unexpected changes

[14]Innovations were estimated using an AR(1) model for both output and money supply. Seasonally adjusted M3 money supply is taken from the IMF database, code: USI59MCCB. We use quarter averages from monthly data in order to ensure comparability with GDP, which is a flow variable.

[15]Empirical estimates of γ reported in the literature are quite imprecise and do not provide evidence against a relatively low degree of risk aversion. Brown and Gibbons (1985) estimate a range of γ between 0.09 and 7. Mankiw, Rotemberg and Summers (1985) estimate values between 0.09 and 0.51 when utility is separable between consumption and leisure. Miron (1986) estimates a range between 0.02 and 1.71. Harvey (1988) estimates a range between 0.33 and 0.96 with seasonally adjusted data.

in productivity or money supply. In the extreme case of full price flexibility, $\alpha \rightarrow 0$, inflation and output growth from equations (20) and (21) converge to $\Delta y_t = \mu_x + \varepsilon_{x,t}$ and $\Delta p_t = \mu + \varepsilon_{m,t} - \varepsilon_{x,t}$. Hence, output and prices behave like random walks.[16] A positive productivity shock at time t leads to an instantaneous proportional increase in output and an instantaneous proportional decline in prices. Similarly, a positive monetary shock leads to an instantaneous upward adjustment of prices but no change in output. However, neither shock changes agents' expectations about future output and prices, since the variables are unpredictable. As a result, there are no intertemporal substitution effects between current and future consumption and expected real rates remain constant. This together with a constant expected inflation leave the slope of the yield curve unchanged.

In the other extreme case of complete price stickiness, that is, when $\alpha \rightarrow 1$, equations (20) and (21) suggest that inflation is constant, $\Delta p_t = \mu$, and output behaves like a random walk, $\Delta y_t = \mu_x + \varepsilon_{m,t}$. As a result, future output and consumption are unforecastable and, hence, economic shocks to current output do not affect the slope of the yield curve since they do not affect expected real rates and inflation.

5 Can the Model Explain the Joint Price and Output Predictability?

5.1 Adjusting the Model for Coupon Carrying Bonds

In the theoretical model developed above, we presented solutions of the term structure of nominal zero-coupon bonds. However, in the empirical section below, we estimate the model using yields to maturity of coupon-carrying bonds. In this subsection, we adjust the asset pricing model to fit data on coupon-carrying bonds.

Our approximation is based on an accounting identity which links the yield to maturity of a $\tau-$period coupon-carrying bond with the one-period holding-period returns. Based on this accounting identity, the log yield to maturity, $r(\tau)_t$, of a $\tau-$period coupon-carrying bond is a weighted average

[16] Note, however, that the presence of supply shocks induces a negative correlation between output growth and inflation. Hence, the levels of output and consumer prices are not independent random walks, but contain a common stochasttic trend.

of future expected log gross returns on a 1-period bond, $E_t r(1)_{t+i}$:

$$r(\tau)_t = \sum_{i=0}^{\tau-1} w_i E_t r(1)_{t+i} \tag{27}$$

where the weights w_i sum up to unity and decline monotonically over time as follows: $w_i = g^i(1-g)/(1-g^\tau)$, $g = 1/(1+\overline{r(\tau)})$ and $\overline{r(\tau)}$ is the sample mean of $r(\tau)_t$.[17] The declining weighting scheme of future 1-period returns in equation (27) reflects the fact that the long-term bond carries coupons and coupons of the near future have a higher present value than coupons of the distant future.

Using equations (15), (20), (21) and (27), we obtain after some algebra:[18]

$$s_{\tau,t} = (\kappa(\tau,g) - \alpha)(\gamma - 1)\psi(L)\varepsilon_t \tag{28}$$

where $\kappa(\tau,g) = \frac{(1-g)\alpha(1-(\alpha g)^\tau)}{(1-g^\tau)(1-\alpha g)}$. Equation (28) is the counterpart of equation (25) for coupon carrying bonds.

5.2 Single Equation Estimates

Equations (23) and (28) suggest that the forecasting relationships between the yield spread and future output and price growth are completely described by five parameters: α, γ, τ, h, g. When regressing future output and price growth on the yield spread, the latter three parameters are known by the econometrician. For instance, with 10-year quarterly bond yield data, $\tau = 40$ and $g = \frac{1}{1+(1/4)\overline{r(40)}}$, where $\overline{r(40)}$ is the sample mean of the 10-year bond yield. In our sample, $\overline{r(40)} = 0.0717$, hence $g = 0.9824$. Similarly, h can be fixed at a specific value. In the empirical analysis, we estimate the model for a range of forecast horizons, h, between two and twelve quarters. The remaining two parameters α and γ can be estimated from equations (20), (23) and (28).

The degree of price stickiness, α, can be estimated from equation (20). Rewrite (20) in the following way:

$$\Delta p_t = \mu + \alpha \Delta p_{t-1} + v_t \tag{29}$$

where $v_t = (1-\alpha)\varepsilon_t$ is a white noise process related to the structural shock ε_t. The null hypothesis of a unit root in inflation corresponds to the

[17] See Campbell et.al. (1997), pp. 407-409, for a derivation.

[18] See Appendix B for a derivation.

case of full price stickiness ($\alpha = 1$). Augmented Dickey-Fuller (ADF) tests for a unit root in the data are reported in the first four columns of Table 4. The results of these tests confirm the typical finding that GDP growth and the yield spread are stationary, whereas quarterly inflation contains a unit root.

It is well known that ADF tests are biased towards nonrejection of the unit-root hypothesis if the series contains a structural break. In order to account for a structural break with unknown timing in the inflation series, we applied the sequential ADF test of Zivot and Andrews (1992) to test for a unit root in inflation against the alternative hypothesis that the series is trend-stationary with a structural break of unknown timing in the mean and the drift rate of the deterministic trend.

The sequential ADF equations have been estimated with up to eight lagged changes of the series in order to account for serial correlation in the residuals. Column $Inf\ t(\tau)$ of Table 4 reports the results of the sequential ADF tests with a structural break in the deterministic component of the series. The unit root hypothesis can be rejected against the alternative of stationarity with a shift in the mean and the drift rate of the deterministic trend, confirming the results of Malliaropulos (2000). The break point is estimated at 1981:Q3, shortly before the move to a monetary policy regime of inflation targeting had been completed in 1982. Figure 4 plots quarterly inflation along with its fitted trend and the adjusted series after removing the segmented trend.

Table 5 reports estimation results of the degree of price stickiness when the structural break in the deterministic part of inflation is taken into account. The estimated inflation equation is:

$$\Delta p_t = b_0 + \alpha \Delta p_{t-1} + b_1 t + b_2 D_t + b_3 D_t t + v_t \tag{30}$$

where D_t is a dummy variable which takes the value 0 before 1981:Q3 and unity afterwards. Our point estimate of α is 0.52 and is very precisely estimated with more than eight standard deviations from zero, suggesting a moderate degree of price stickiness.

Next, given our estimate for α, we can use either equation (23) or equation (28) to estimate γ. This, however, presupposes that we can extract a time series for the underlying shock ε_t from the data. In order to avoid measurement errors of innovations in productivity and money supply, we substitute equation (23) for $\psi(L)\varepsilon_t$ in equation (28), obtaining:

$$\frac{1}{h}E_t(y_{t+h} - y_t) = a_0 + \frac{\kappa(h)}{(\kappa(\tau, g) - \alpha)(\gamma - 1)} s_{\tau,t} \tag{31}$$

$$\frac{1}{h}E_t(p_{t+h} - p_t) = \beta_0 - \frac{\kappa(h)}{(\kappa(\tau, g) - \alpha)(\gamma - 1)} s_{\tau,t} \tag{32}$$

where a_0, β_0 are two constants.[19]

In order to obtain two regression equations from (31), (32), define two error terms $\eta_{t+h} \equiv \frac{1}{h}(y_{t+h} - y_t) - \frac{1}{h}E_t(y_{t+h} - y_t)$ and $\xi_{t+h} \equiv \frac{1}{h}(p_{t+h} - p_t) - \frac{1}{h}E_t(p_{t+h} - p_t)$. Then, we can rewrite (31), (32) as two regression equations:

$$\frac{1}{h}(y_{t+h} - y_t) = a_0 + \frac{\kappa(h)}{(\kappa(\tau, g) - \alpha)(\gamma - 1)} s_{\tau,t} + \eta_{t+h} \tag{33}$$

$$\frac{1}{h}(p_{t+h} - p_t) = \beta_0 - \frac{\kappa(h)}{(\kappa(\tau, g) - \alpha)(\gamma - 1)} s_{\tau,t} + \xi_{t+h} \tag{34}$$

In general, the time $t+h$ forecast errors, η_{t+h}, ξ_{t+h}, will not be correlated with the time t spread, $s_{\tau,t}$, so Non-Linear Least Squares is an appropriate estimation method. Table 6 reports NLLS estimates of equations (33), (34). In order to test for robustness of our results, we estimate the regressions for a wide range of forecast horizons, h, between two and twelve quarters ahead. In order to account for autocorrelation in the residuals due to overlapping observations, we computed Newey and West standard errors, which are robust to serial correlation and heteroskedasticity. Panel A of the table reports estimation results of the output forecast regression, (33). The estimates of γ at forecast horizons between four and twelve quarters range between 0.43 and 0.59 and are all significantly different from zero. The adjusted $R^{2\prime}s$ are similar to the $R^{2\prime}s$ of the unrestricted OLS regressions in Table 2, Panel C, suggesting that imposing the restrictions of the model does not reduce the predictive power of the yield spread.

Panel B of the table reports estimation results of the price forecast regression, (34). If the yield spread is a symmetric predictor of output and prices,

[19] Adding a constant term in equations (31), (32) is justified by our assumption that money supply and productivity follow random walks with drift – see equations (17), (18). We skipped these constants from equation (22) in order to simplify the notation.

then estimates of γ from (34) should be not significantly different from estimates of γ from the output forecast regression, (33). The point estimates of γ from the price forecast regression range between 0.23 and 0.52 but are less precisely estimated than in the output forecast regression. The third row of the table reports Wald tests of the null hypothesis: $\widehat{\gamma} = \gamma_0$, where γ_0 is the estimate of γ from the output forecast regression. The null hypothesis cannot be rejected at any horizon, suggesting that our model can explain the symmetric predictive power of the yield spread for output and prices.

5.3 System Estimates

Estimating equations (33) and (34) by single equation methods may lead to inefficient estimates of γ, since α is treated as a pre-fixed parameter in the second stage regressions, (33) and (34). As a result, the standard errors of the coefficient γ in Table 7 are likely understated since their calculation does not take into account the sampling variation in α. In order to account for this effect, we estimate (30), (33), (34) as a system of nonlinear, seemingly unrelated regressions. Since our model predicts that both inflation and output are driven by the same innovations, nonlinear SUR is efficient because it accounts for the cross-correlation in the residuals.

Table 7 reports the SUR estimation results. In order to test the ability of our model to explain the symmetric predictive power of the yield spread for output and inflation, we estimate γ twice: from both the predictive regression for output and from the predictive regression for prices. We then test the restriction that these two estimates of γ are equal using a Wald test. Our estimates of γ range between 0.28 at $h = 4$ and 0.68 at $h = 12$ and are quite precisely estimated, especially as the forecast horizon lengthens. Both predictive regressions imply remarkably similar estimates of γ. In fact, the Wald test reported in the last row of the table cannot reject the restriction that γ is equal across both predictive regressions at any forecast horizon.

Table 8 reports estimation results of the restricted system of equations with γ resricted to be equal across both predictive regressions. Estimates of α range between 0.38 and 0.47 with more than three standard errors from zero. Again, with the exception of $h = 2$, point estimates of γ range between 0.33 and 0.57 and are still precisely estimated with more than two standard errors from zero.

6 Concluding Remarks

We examined the predictive ability of the yield spread for future inflation and discovered that the current yield spread is negatively related to the future *level* of inflation for horizons between one quarter and one and a half years ahead. Moreover, the predictability of inflation is symmetric to the predictability of output.

In order to explain this new stylized fact, we developed a parsimonious monetary consumption based asset pricing model, whose main innovation is the introduction of nominal rigidities in the economy in form of sticky prices of the consumption good. Due to price stickiness, shocks to the economy generate predictable changes in future output and prices, hence, allowing for intertemporal consumption smoothing effects on interest rates. This generates a correlation between the current yield spread and future expected output growth and inflation. We derived analytic solutions of the model, which relate output growth, inflation and the term structure to unanticipated changes in productivity and money supply.

In the model, productivity shocks in excess of shocks to the money supply generate a positive correlation between the yield spread and future output growth and a negative correlation between the yield spread and future inflation. The theoretical model can explain the observed stylized facts, provided that there exists some price stickiness and that the elasticity of intertemporal substitution is larger than unity.

The model was subsequently estimated and produced reasonable parameter values. These estimates suggest that the model can account for the observed symmetric predictability of future output growth and inflation.

Despite its simplicity, the model goes a long way in explaining the stylized facts. However, it is clear that such a model is too restrictive to account for the observed magnitude of the correlations between the variables. In the model, the term premium is constant and the yield spread is perfectly correlated with future output growth and inflation. This is because all variables are driven by the same stochastic disturbance, namely the innovation of productivity in excess of money supply.

In order to account for less than perfect correlation between the variables, one has to include more than one stochastic disturbance driving the variables. One possibility would be to relax the assumption of a constant velocity of money in order to allow for asymmetric effects of productivity and monetary shocks on expected output growth and inflation.[20] Another possibility would be to allow for a time-varying term premium, which affects the slope of the yield curve independently of productivity and money supply shocks, leading to changes in the yield spread which are uncorrelated with predictable changes in future output and prices.[21]

A further modification of the model would be to introduce richer dynamics in the driving processes of money supply and productivity. For example, assuming an AR(1) process for money growth allows for predictable changes in output and inflation due to mean-reversion in money supply in addition to predictable changes related to price stickiness.[22] Allowing for richer dynamics of the stochastic disturbances driving the economy will lead to more flexibility in the dynamic adjustment of the term structure to economic shocks.

[20] This could for example account for the observation that, during the 1960s, the yield spread had stronger predictive power for inflation than for output growth (see Panel D of Tables 2 and 3).

[21] A decline in the term premium due to lower output volatility might be able to explain the breakdown of the predictive power of the yield spread during the 1990s.

[22] See Rotemberg (1996), equations (12)-(16), for a modification of the model in this direction.

Appendix A: Solution of the Model

Note that, because the cash-in-advance constraint is binding, $P_{t-1}C_{i,t-1} = M_{i,t-1}$, equation (14) simplifies to:

$$M_{i,t} = P_{i,t-1}Y_{i,t-1} + T_{i,t} + \sum_{\tau=1}^{n} R_{\tau,t-\tau}B_{\tau,t-\tau} - \sum_{\tau=1}^{n} B_{\tau,t} \tag{A.1}$$

Then, using equations (10) and (13), we can rewrite equation (A.1) as:

$$C_{i,t} = \frac{P_{i,t-1}}{P_t}Y_{t-1}\left(\frac{P_{t-1}}{P_{i,t-1}}\right)^{\frac{1}{1-q}} + \frac{T_{i,t}}{P_t} + \sum_{\tau=1}^{N} R_{\tau,t-\tau}\frac{B_{\tau,t-\tau}}{P_t} - \sum_{\tau=1}^{N}\frac{B_{\tau,t}}{P_t} \tag{A.2}$$

Substituting equations (10), (11) and (A.1) into (12), we obtain:

$$\begin{aligned} U_{i,t} \quad = \quad & E_t\sum_{k=0}^{\infty}\beta^k\frac{1}{1-\gamma}[\frac{P_{i,t+k-1}}{P_{t+k}}Y_{t+k-1}\left(\frac{P_{t+k-1}}{P_{i,t+k-1}}\right)^{\frac{1}{1-q}} \\ & +\frac{T_{i,t+k}}{P_{t+k}} + \sum_{\tau=1}^{N} R_{\tau,t+k-\tau}\frac{B_{\tau,t+k-\tau}}{P_{t+k}} - \sum_{\tau=1}^{N}\frac{B_{\tau,t+k}}{P_{t+k}}]^{1-\gamma} \\ & -E_t\sum_{k=0}^{\infty}\beta^k\psi X_{t+k}^{1-\gamma}\frac{1}{1-n}\left(\frac{Y_{t+k}}{X_{t+k}}\right)^{1-n}\left(\frac{P_{t+k}}{P_{i,t+k}}\right)^{\frac{1-n}{1-q}} \\ & -E_t\sum_{k=0}^{\infty}\beta^k\frac{c}{2}X_{t+k}^{1-\gamma}[\ln P_{i,t+k} - \ln P_{i,t+k-1}]^2 \end{aligned} \tag{A.3}$$

Maximizing (A.3) w.r.t. $B_{\tau,t}$ and $P_{i,t}$ leads to the following optimality conditions, evaluated at the symmetric equilibrium where $P_{i,t} = P_t$ and $Y_{i,t} = Y_t$:

$$E_t\left[\beta^\tau\left(\frac{C_{t+\tau}}{C_t}\right)^{-\gamma}\frac{P_t}{P_{t+\tau}}R_{\tau,t}\right] = 1, \qquad \tau = 1, ..., N \tag{A.4}$$

$$\begin{aligned} 0 \quad = \quad & E_t[-\beta\frac{q}{1-q}\frac{P_t}{P_{t+1}}\left(\frac{M_t}{P_tX_t}\right)^{1-\gamma} + \psi\frac{1}{1-q}\left(\frac{M_t}{P_tX_t}\right)^{1-n} \\ & +c\left(\ln P_t - \ln P_{t-1}\right) - \beta c\left(\frac{X_{t+1}}{X_t}\right)^{1-\gamma}\left(\ln P_{t+1} - \ln P_t\right)] \end{aligned} \tag{A.5}$$

Equation (A.4) is the well-known asset pricing formula of Consumption-CAPM.

To understand how the term structure is related to expectations of consumption growth and inflation, we assume that the joint conditional distribution of consumption and consumption prices is i.i.d. lognormal. Thus, defining the continuously compounded, annualized yield to maturity at time t on a nominal discount bond with term τ as $r_t(\tau) = \log(R_{\tau,t})/\tau$ and taking logs of equation (A.4) leads to equation (15) in the text.

We log-linearize equation (A.5) around the sample means of P_{t+1}/P_t, M_t/P_tX_t, X_{t+1}/X_t. Denoting these sample means $1+\pi$, M/PX, $1+g$ and ignoring constants, the loglinearized version of equation (A.5) reads:

$$\begin{gathered} E_t[\beta\left(-c(1+g)^{1-\gamma} + \tfrac{q}{1-q}\tfrac{(M/PX)^{1-\gamma}}{(1+\pi)}\right)(p_{t+1}-p_t) + c(p_t - p_{t-1}) \\ + \left(\psi\tfrac{1-n}{1-q}(M/PX)^{1-n} - \beta(1-\gamma)\tfrac{q}{1-q}\tfrac{(M/PX)^{1-\gamma}}{(1+\pi)}\right)(m_t - p_t - x_t) \\ -\beta c\pi(1-\gamma)(1+g)^{1-\gamma}(x_{t+1}-x_t)] = 0 \end{gathered} \tag{A.6}$$

where lowercase letters, p_t, m_t, x_t denote logs of the upper case variables.

Equation (A.6) is a second-order difference equation in p_t. As in Rotemberg (1982, 1986), this equation has a unique, nonexplosive solution if one of the two roots of the characteristic equation is smaller than one while the other is larger than one. Thus, the solution to equation (A.6) is equation (16) in the text, where α is the root smaller than one and $1/\delta$ is the other root of the characteristic equation.

Appendix B: Adjusting the Model for Coupon-Carrying Bonds

Using equation (15) to compute the one-period nominal interest rate, we obtain:

$$r(1)_t = -\log(\beta) + \gamma E_t(c_{t+1} - c_t) + E_t(p_{t+1} - p_t) + \theta \tag{B.1}$$

Computing future expected one-period interest rates from equation (B.1) and substituting into equation (27), we obtain:

$$\begin{aligned} r(\tau)_t &= \sum_{i=0}^{\tau-1} w^i \left[\gamma E_t(c_{t+i+1} - c_{t+i}) + E_t(p_{t+i+1} - p_{t+i})\right] \\ &\quad - \log(\beta) + \theta \end{aligned} \tag{B.2}$$

Hence, the yield to maturity of a $\tau-$period coupon bond can be expressed as a weighted average of expected future consumption growth and inflation plus a constant term, $-\log(\beta)+\theta$, which reflects consumers' time preferences and term premia.

In order to obtain a closed form solution of equation (B.2) in terms of observables, similar to equation (24), we compute from equations (21) and (20) the conditional expectation of the growth rate of output (consumption) and prices from period $t+i$ to period $t+i+1$ for $i \geq 0$:

$$E_t(c_{t+i+1} - c_{t+i}) = -E_t(p_{t+i+1} - p_{t+i}) = \alpha^{i+1}\psi(L)\varepsilon_t \tag{B.3}$$

Substituting equation (B.3) into equation (B.2), we obtain after some algebra as the counterpart of equation (24) for coupon carrying bonds:

$$r(\tau)_t = -\log(\beta) + \kappa(\tau, g)(\gamma - 1)\psi(L)\varepsilon_t + \theta \tag{B.4}$$

where $\kappa(\tau, g) = \frac{(1-g)\alpha(1-(\alpha g)^\tau)}{(1-g^\tau)(1-\alpha g)}$. Note that $\kappa(1, g) = \alpha$, implying that equation (B.4) can be used to determine the whole term structure of interest rates as a function of time t shocks ε_t.

Using equation (B.4) to compute $r(1)_t$ and subtracting the resulting equation from (B.4), we obtain for the yield spread equation (28) in the text.

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Table 1: Descriptive Statistics
(Sample: Quarterly data 1960:Q1-2003:Q2).
Panel A: Summary Statistics

variable	mean	variance	skewness	kurtosis
GDP growth	3.27	5.21	-0.44	0.15
Inflation	4.36	7.94	1.33	1.32
3-m T-bill	5.76	7.20	1.12	1.72
3-y yield	6.78	7.20	0.98	1.01
5-y yield	6.98	6.78	0.99	0.86
10-y yield	7.17	6.41	0.94	0.58
3y-3m spread	1.02	0.62	0.01	0.10
5y-3m spread	1.21	0.96	-0.10	-0.35
10y-3m spread	1.40	1.44	-0.12	-0.62

Panel B: Correlations

	GDP growth	Inflation	3-y spread	5-y spread	10-y spread
GDP growth	1.00	-0.39	0.40	0.42	0.44
Inflation		1.00	-0.32	-0.35	-0.36
3y-3m spread			1.00	0.98	0.94
5y-3m spread				1.00	0.99
10y-3m spread					1.00

Notes: GDP growth is the four-quarter ahead difference of log GDP. Inflation is the four-quarter ahead difference of log CPI. 3-m T-Bill is the 3-month Treasury Bill rate. Bond yields are yields to maturity of 3-year, 5-year, and 10-year Treasury bonds. All data are quarerly averages and all yields are annualized. Yield spreads are calculated over 3-month T-Bill rates.

Table 2: Predicting k - quarter ahead annualized real GDP growth with the yield spread

$$(y_{t+k} - y_t)100(\tfrac{4}{k}) = a_0 + a_1 s_t + u_t$$

Panel A: s_t : 3-year spread (sample 1960:1 - 2003:2)

	Quarters ahead (k)									
	1	2	3	4	5	6	7	8	12	16
a_1	1.04* (2.65)	1.22* (2.99)	1.25* (3.26)	1.23* (3.38)	1.21* (3.61)	1.17* (3.70)	1.08* (3.62)	0.97* (3.748)	0.60* (2.769)	0.30 (1.64)
$\overline{R}^2$	0.05	0.12	0.16	0.19	0.22	0.23	0.23	0.22	0.15	0.06

Panel B: s_t : 5-year spread (sample 1960:1 - 2003:2)

	Quarters ahead (k)									
	1	2	3	4	5	6	7	8	12	16
a_1	0.85* (3.09)	1.02* (3.25)	1.03* (3.38)	1.02* (3.32)	1.00* (3.38)	0.96* (3.38)	0.90* (3.31)	0.80* (3.23)	0.49* (2.70)	0.25 (1.84)
$\overline{R}^2$	0.06	0.13	0.17	0.20	0.23	0.24	0.24	0.23	0.15	0.07

Panel C: s_t : 10-year spread (sample 1960:1 - 2003:2)

	Quarters ahead (k)									
	1	2	3	4	5	6	7	8	12	16
a_1	0.74* (3.37)	0.86* (3.20)	0.87* (3.20)	0.85* (3.10)	0.82* (3.03)	0.77* (2.94)	0.74[1]* (2.80)	0.63* (2.68)	0.35* (2.03)	0.17 (1.46)
$\overline{R}^2$	0.06	0.13	0.16	0.19	0.21	0.21	0.20	0.18	0.08	0.03

Panel D: Predicting one year ahead GDP growth ($k = 4$, s_t : 10-year spread)

	1960:1969	1970:1979	1980:1989	1990:2003
a_1	1.86* (2.28)	1.92* (7.12)	1.12* (5.15)	0.23 (0.64)
$\overline{R}^2$	0.17	0.72	0.42	0.01

Notes: y_t : log real GDP, s_t: yield spread over 3-month T-Bill. t-statistics in parentheses below coefficient estimates are based on Newey and West heteroskedasticity and autocorrelation consistent standard errors up to $k-1$ lags. * denotes significance at the 5% level.

Table 3: Predicting k - quarter ahead annualized consumer price inflation with the yield spread

$$(p_{t+k} - p_t)100(\tfrac{4}{k}) = b_0 + b_1 s_t + u_t$$

Panel A: s_t : 3-year spread (sample 1960:1 - 2003:2)

	Quarters ahead (k)									
	1	2	3	4	5	6	7	8	12	16
b_1	-1.45* (−2.92)	−1.39* (−2.45)	−1.31* (−2.17)	−1.22* (−2.02)	−1.12 (−1.92)	−0.99 (−1.80)	−0.85 (−1.70)	−0.74 (−1.62)	−0.48 (−1.53)	−0.33 (−1.36)
t_{c_1}	-0.88	-0.38	-0.13	0.00	0.17	0.32	0.46	0.52	0.41	-0.15
$\overline{R}^2$	0.15	0.15	0.14	0.13	0.11	0.09	0.07	0.05	0.02	0.01

Panel B: s_t : 5-year spread (sample 1960:1 - 2003:2)

	Quarters ahead (k)									
	1	2	3	4	5	6	7	8	12	16
b_1	−1.28* (−3.33)	−1.21* (−2.74)	−1.15* (−2.41)	−1.07* (−2.21)	−0.99* (−2.09)	−0.89* (−1.96)	−0.77 (−1.84)	−0.67 (−1.74)	−0.44 (−1.67)	−0.31 (−1.73)
t_{c_1}	-1.23	-0.58	-0.30	-0.14	0.02	0.15	0.27	0.33	0.18	-0.36
$\overline{R}^2$	0.18	0.18	0.17	0.15	0.13	0.11	0.08	0.06	0.03	0.01

Panel C: s_t : 10-year spread (sample 1960:1 - 2003:2)

	Quarters ahead (k)									
	1	2	3	4	5	6	7	8	12	16
b_1	−1.06* (−3.30)	−0.98* (−2.66)	−0.92* (−2.33)	−0.86* (−2.11)	−0.80* (−1.96)	−0.71 (−1.85)	−0.62 (−1.73)	−0.53 (−1.63)	−0.33 (−1.64)	−0.28* (−1.96)
t_{c_1}	−1.16	-0.43	-0.15	-0.02	0.08	0.17	0.26	0.30	0.08	-0.56
$\overline{R}^2$	0.17	0.16	0.14	0.13	0.11	0.09	0.07	0.05	0.02	0.01

Panel D: Predicting one year ahead inflation ($k = 4$, s_t : 10-year spread)

	1960:1969	1970:1979	1980:1989	1990:2003
a_1	−2.13 (−2.94)	−1.66* (−4.72)	−1.07* (−5.06)	0.02 (0.21)
t_{c_1}	0.63	0.65	0.13	-0.42
$\overline{R}^2$	0.43	0.44	0.47	-0.02

Notes: p_t : log of CPI, s_t: yield spread. t-statistics in parentheses below coefficient estimates are based on Newey and West heteroskedasticity and autocorrelation consistent standard errors up to $k-1$ lags. Row t_{c_1} reports t-statistics of the null hypothesis that the yield spread predicts opposite changes in real GDP and prices, i.e. $b_1 = -a_1$. * denotes significance at the 5% level.

Table 4: **T**ests for Unit Roots

	ADF tests				Zivot and Andrews tests		
	with constant		with constant + trend		$Inf\ t(\tau)$		Time of break
	$k=4$	$k=8$	$k=4$	$k=8$	$k=4$	$k=8$	
y_t	-1.78	-1.42	-4.06*	-3.15	–	–	–
p_t	-1.83	-1.63	-1.46	-1.16	–	–	–
$s_{3,t}$	-4.28*	-3.48*	-4.40*	-3.65*	–	–	–
$s_{5,t}$	-3.57*	-3.47*	-4.16*	-3.87*	–	–	–
$s_{10,t}$	-3.57*	-3.47*	-3.93*	-3.96*	–	–	–
Δy_t	-5.64*	-4.04*	-5.81*	-4.22*	–	–	–
Δp_t	-2.02	-1.86	-2.36	-2.28	-5.40*	-5.30*	1981:Q3

Notes: y_t : log real GDP, p_t: log CPI, $s_{3,t}$: Yield spread between 3-year Treasury bond and 3-month Treasury bill, $s_{5,t}$: Yield spread between 5-year Treasury bond and 3-month Treasury bill, $s_{10,t}$: Yield spread between 10-year Treasury bond and 3-month Treasury bill, k: number of lags in ADF regression. Critical values of ADF test at the 5% significance level: -2.86 (with constant), -3.41 (with constant and trend). Critical value of Zivot and Andrews test at the 5% significance level: -5.08. *: significant at the 5% level.

Table 5: Estimates of inflation equation (30)

$$\Delta p_t = b_0 + \alpha \Delta p_{t-1} + b_1 t + b_2 D_t + b_3 D_t t + v_t$$

(Estimates by OLS, sample: 1960:1-2003:2).

b_0	α	b_1	b_2	b_3	$\overline{R}^2$
-0.44 (−3.42)	0.52^{**} (8.45)	0.015^{**} (6.21)	1.20^{**} (3.66)	-0.018^{**} (−5.51)	0.77

Notes: Δp_t is quarterly consumer price inflation. D_t is a dummy variable which takes the value 0 before 1981:Q3 and unity afterwards. t-statistics are reported in parentheses below coefficient estimates. *(**) denote significance at the 5% (1%) level.

Table 6: Single equation estimates of output and price forecast regressions (33), (34)
(**E**stimates by Nonlinear Least Squares, sample:1960:1-2003:2)

Panel A: **O**utput forecast regression

$$\frac{400}{h}(y_{t+h} - y_t) = a_0 + \frac{\kappa(h)}{(\kappa(40,g)-\alpha)(\gamma-1)} s_{40,t} + \eta_{t+h}$$

where $\kappa(h) = \frac{\alpha(1-\alpha^h)}{h(1-\alpha)}$, $\kappa(40, g) = \frac{(1-g)\alpha(1-(\alpha g)^{40})}{(1-g^{40})(1-\alpha g)}$, $g = \frac{1}{1+(1/4)\overline{r(40)}} = 0.9824$

	Forecast horizon (h)					
	2	4	6	8	10	12
a_0	2.04** (4.04)	2.10** (3.70)	2.25** (4.02)	2.45** (4.52)	2.65** (5.14)	2.82** (5.77)
γ	0.11 (0.46)	0.43** (2.42)	0.56** (3.87)	0.59** (3.91)	0.56** (3.24)	0.50* (2.02)
$\overline{R}^2$	0.13	0.19	0.21	0.18	0.13	0.08

Panel B: Price forecast regression

$$\frac{400}{h}(p_{t+h} - p_t) = \beta_0 - \frac{\kappa(h)}{(\kappa(40,g)-\alpha)(\gamma-1)} s_{40,t} + \xi_{t+h}$$

where $\kappa(h) = \frac{\alpha(1-\alpha^h)}{h(1-\alpha)}$, $\kappa(40, g) = \frac{(1-g)\alpha(1-(\alpha g)^{40})}{(1-g^{40})(1-\alpha g)}$, $g = \frac{1}{1+(1/4)\overline{r(40)}} = 0.9824$

	Forecast horizon (h)					
	2	4	6	8	10	12
β_0	5.61** (8.23)	5.45** (5.87)	5.27** (5.17)	5.03** (5.02)	4.88** (5.10)	4.82** (5.42)
γ	0.23 (0.97)	0.44 (1.77)	0.52* (2.00)	0.51 (1.80)	0.46 (1.71)	0.44 (1.52)
$\overline{R}^2$	0.16	0.13	0.09	0.05	0.03	0.02
$Wald$ [$p-value$]	0.24 [0.62]	0.00 [0.97]	0.02 [0.89]	0.06 [0.80]	0.06 [0.80]	0.02 [0.87]

Notes: $\frac{400}{h}(y_{t+h} - y_t)$ is the h-quarter ahead annualized real GDP growth; $\frac{400}{h}(p_{t+h} - p_t)$ is the h-quarter ahead annualized consumer price inflation; $s_{40,t}$ is the 10-year - 3-month yield spread. $\overline{r(40)}$ is the sample mean of the 10-year bond yield. t-statistics in parentheses below coefficient estimates (based on Newey and West heteroskedasticity and autocorrelation consistent standard errors up to $h-1$ lags). *(**) denote significance at the 5% (1%) level. "$Wald$" is the Wald test of the null hypothesis that the estimate of γ in the price regression is equal to the point estimate of γ in the output regression, reported in Panel A. The test statistic is $\chi^2(1)$ distributed. The marginal significance level of the test is reported in square brackets below the test statistic.

Table 7: System estimates of output, price and inflation equations (33), (34), (30)

$$\frac{400}{h}(y_{t+h} - y_t) = a_0 + \frac{\kappa(h)}{(\kappa(40,g)-\alpha)(\gamma_1-1)} s_{40,t} + \eta_{t+h}$$
$$\frac{400}{h}(p_{t+h} - p_t) = \beta_0 - \frac{\kappa(h)}{(\kappa(40,g)-\alpha)(\gamma_2-1)} s_{40,t} + \xi_{t+h}$$
$$\Delta p_t = b_0 + \alpha \Delta p_{t-1} + b_1 t + b_2 D_t + b_3 D_t t + v_t$$

where $\kappa(h) = \frac{\alpha(1-\alpha^h)}{h(1-\alpha)}$, $\kappa(40,g) = \frac{(1-g)\alpha(1-(\alpha g)^{40})}{(1-g^{40})(1-\alpha g)}$, $g = \frac{1}{1+(1/4)\overline{r(40)}} = 0.9824$
(Estimates by Nonlinear SUR, sample:1960:1-2003:2).

	Forecast horizon (h)					
	2	4	6	8	10	12
a_0	2.40** (4.85)	2.41** (4.50)	2.44** (4.57)	2.57** (4.93)	2.72** (5.43)	2.86** (5.92)
β_0	5.22** (8.08)	5.05** (5.82)	5.09** (5.10)	5.03** (5.02)	5.04** (5.12)	5.04** (5.32)
γ_1	−0.15 (−0.38)	0.35* (1.98)	0.50** (3.11)	0.56** (3.24)	0.57** (2.80)	0.55* (2.08)
γ_2	0.00 (0.01)	0.28 (1.36)	0.46* (1.98)	0.55* (2.00)	0.63** (2.75)	0.68** (4.22)
α	0.39** (5.13)	0.41** (4.83)	0.47** (4.98)	0.47** (4.38)	0.45** (3.60)	0.43** (3.37)
b_0	−0.09 (−1.10)	−0.01 (−0.02)	−0.02** (−4.40)	−0.35** (−3.78)	−0.53** (−9.60)	−0.69** (−10.17)
b_1	0.01** (5.42)	0.01** (4.48)	0.01** (4.42)	0.01** (4.83)	0.02** (6.04)	0.02** (7.28)
b_2	0.88* (2.62)	0.76* (2.23)	1.04** (3.39)	1.24** (2.98)	1.68** (5.16)	2.03** (6.04)
b_3	−0.01** (−4.61)	−0.01** (−3.70)	−0.02** (−4.40)	−0.02** (−4.16)	−0.02** (−5.92)	−0.03** (−7.23)
$\overline{R^2}$ output	0.13	0.19	0.21	0.19	0.14	0.09
$\overline{R^2}$ price	0.15	0.12	0.10	0.06	0.04	0.03
$\overline{R^2}$ inflation	0.74	0.74	0.77	0.77	0.76	0.74
$Wald$ [$p-value$]	0.17 [0.68]	0.03 [0.86]	0.02 [0.89]	0.00 [0.96]	0.07 [0.78]	0.70 [0.40]

Notes: See notes in Tables 5 and 6. "$Wald$" is the Wald test of the null hypothesis: $\gamma_1 = \gamma_2$. The test statistic is $\chi^2(1)$ distributed. The marginal significance level of the test is reported in square brackets below the test statistic.

Table 8: Restricted system estimates of output, price and inflation equations (33), (34), (30)

$$\frac{1}{h}(y_{t+h} - y_t) = a_0 + \frac{\kappa(h)}{(\kappa(40,g)-\alpha)(\gamma-1)} s_{40,t} + \eta_{t+h}$$
$$\frac{1}{h}(p_{t+h} - p_t) = \beta_0 - \frac{\kappa(h)}{(\kappa(40,g)-\alpha)(\gamma-1)} s_{40,t} + \xi_{t+h}$$
$$\Delta p_t = b_0 + \alpha \Delta p_{t-1} + b_1 t + b_2 D_t + b_3 D_t t + v_t$$

where $\kappa(h) = \frac{\alpha(1-\alpha^h)}{h(1-\alpha)}$, $\kappa(40,g) = \frac{(1-g)\alpha(1-(\alpha g)^{40})}{(1-g^{40})(1-\alpha g)}$, $g = \frac{1}{1+(1/4)\overline{r(40)}} = 0.9824$
(Estimates by Nonlinear SUR, sample:1960:1-2003:2).

	Forecast horizon (h)					
	2	4	6	8	10	12
a_0	2.35^{**} (4.68)	2.43^{**} (4.34)	2.45^{**} (4.43)	2.58^{**} (4.86)	2.72^{**} (5.42)	2.86^{**} (5.93)
β_0	5.13^{**} (9.20)	5.11^{**} (6.94)	5.15^{**} (6.10)	5.05^{**} (5.67)	4.94^{**} (5.30)	4.85^{**} (4.95)
γ	-0.08 (-0.26)	0.33^{*} (2.00)	0.50^{**} (3.30)	0.56^{**} (3.36)	0.57^{**} (2.73)	0.54^{*} (1.97)
α	0.38^{**} (5.10)	0.41^{**} (4.86)	0.47^{**} (4.98)	0.47^{**} (4.34)	0.45^{**} (3.62)	0.44^{**} (3.42)
b_0	-0.06 (-0.70)	-0.02 (-0.22)	-0.21^{*} (-2.20)	-0.35^{**} (-3.90)	-0.50^{**} (-8.80)	-0.67^{**} (-9.18)
b_1	0.01^{**} (5.23)	0.01^{**} (4.65)	0.01^{**} (4.62)	0.01^{**} (4.90)	0.02^{**} (5.75)	0.02^{**} (6.70)
b_2	0.84^{*} (2.50)	0.79^{*} (2.34)	1.07^{**} (3.50)	1.25^{**} (3.00)	1.62^{**} (4.90)	1.92^{**} (5.50)
b_3	-0.01^{**} (-4.44)	-0.01^{**} (-3.83)	-0.01^{**} (-4.60)	-0.02^{**} (-4.22)	-0.02^{**} (-5.64)	-0.02^{**} (-6.56)
$\overline{R}^2$ output	0.13	0.18	0.21	0.19	0.20	0.14
$\overline{R}^2$ price	0.15	0.13	0.10	0.06	0.08	0.08
$\overline{R}^2$ inflation	0.74	0.74	0.77	0.77	0.81	0.80

Notes: See notes in Tables 5 and 6.

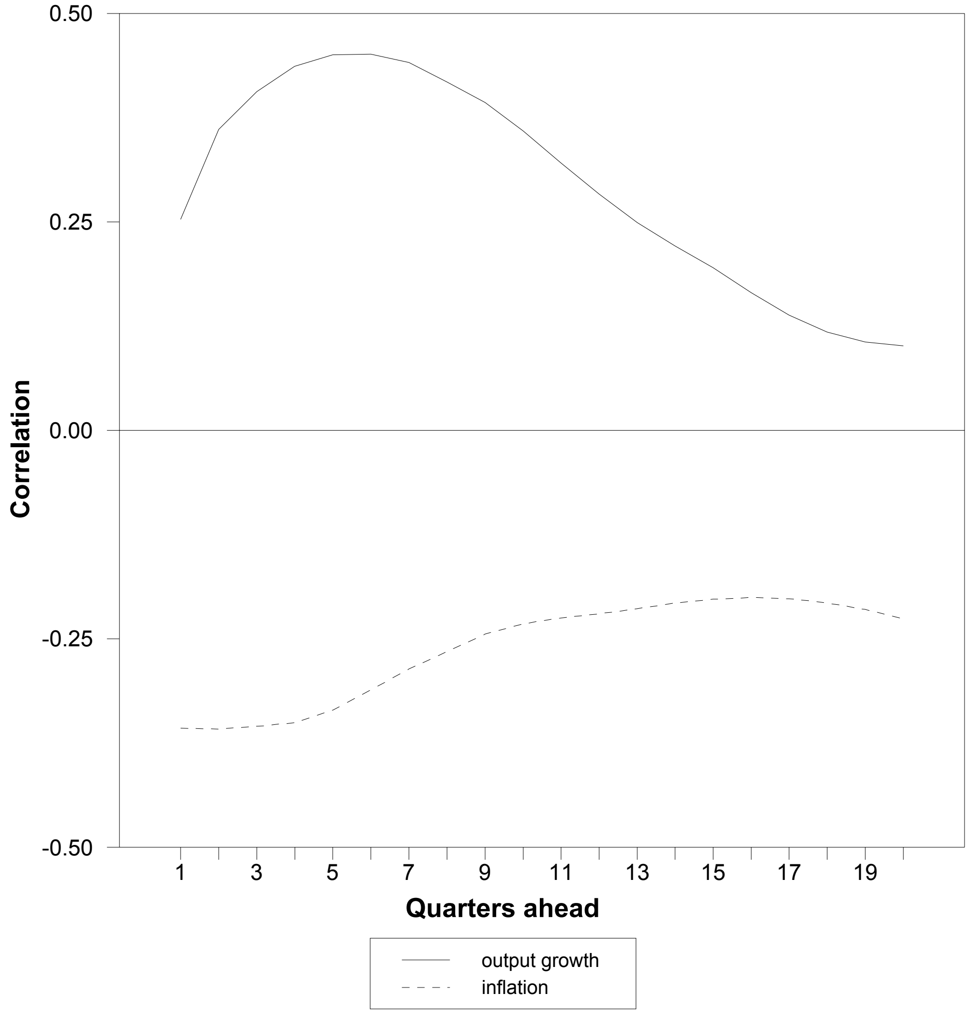

Figure 1: Sample correlations with the current 10-year - 3-month interest rate spread: 'output growth' denotes the correlation of the $h-$quarters ahead annualized real GDP growth. 'inflation' denotes the correlation of the $h-$quarters ahead annualized change in the Consumer Price Index.

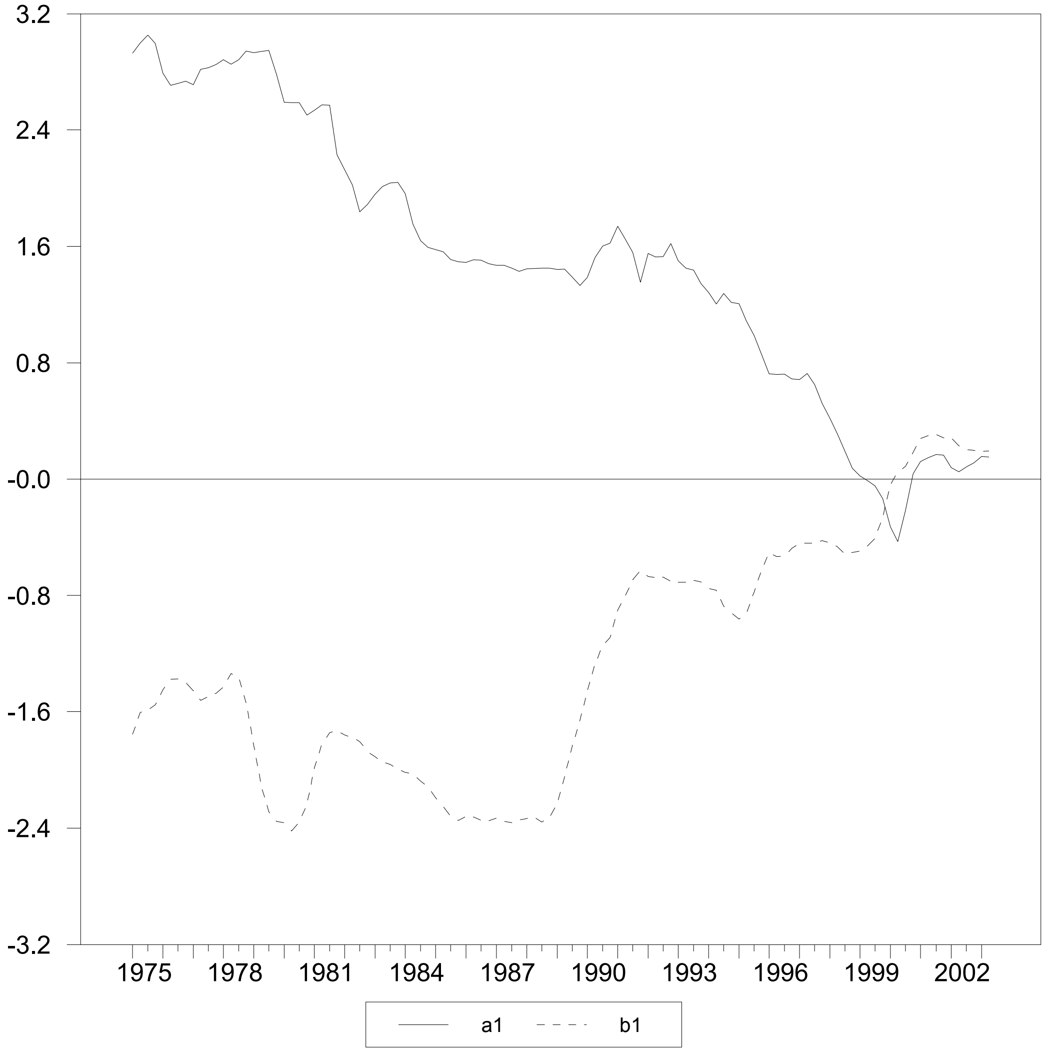

Figure 2: Recursive slope estimates of predictive regressions for one-year ahead real GDP growth (a_1) and inflation (b_1), using a moving window of width = 40 quarters. The regression uses the spread between the 10-year and the 3-month yield.

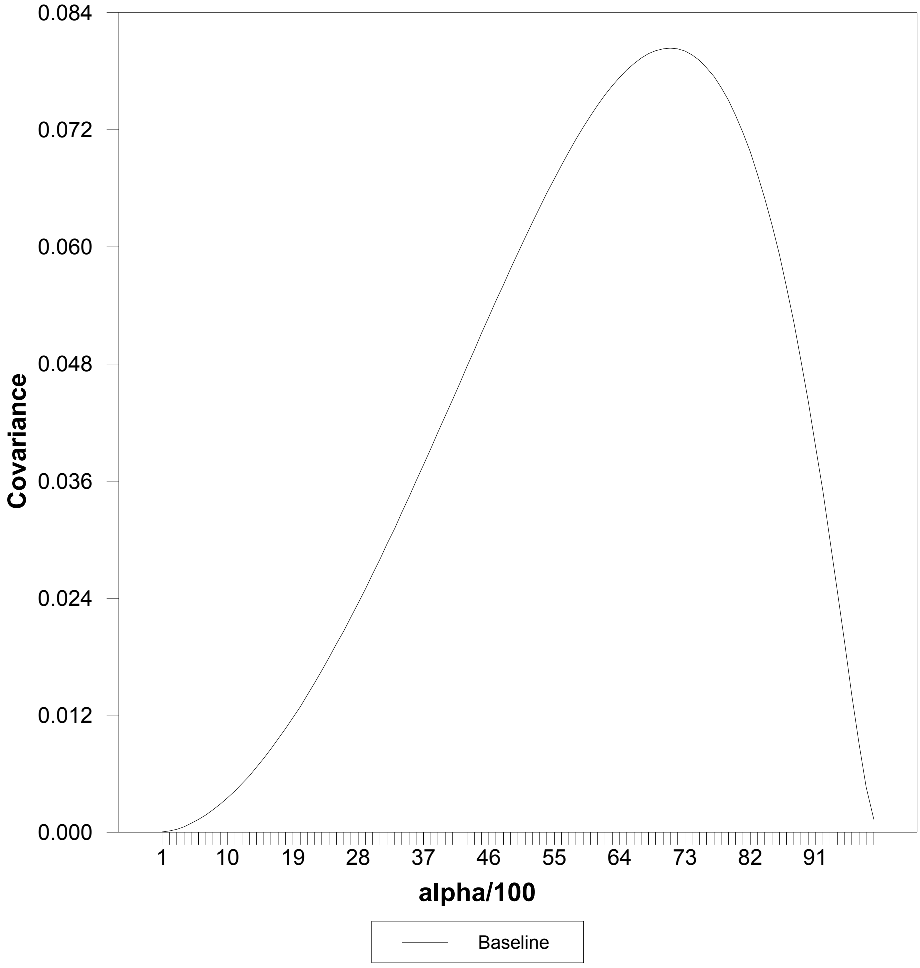

Figure 3: Covariance between the 10-year - 3-month yield spread and h quarter ahead GDP growth. Baseline simulation assumes $n = 40, h = 4, \gamma = 0.6$, $\sigma_{\varepsilon}^{2} = 3.6$. Alpha is the price stickiness parameter, α, and takes values from zero to unity.

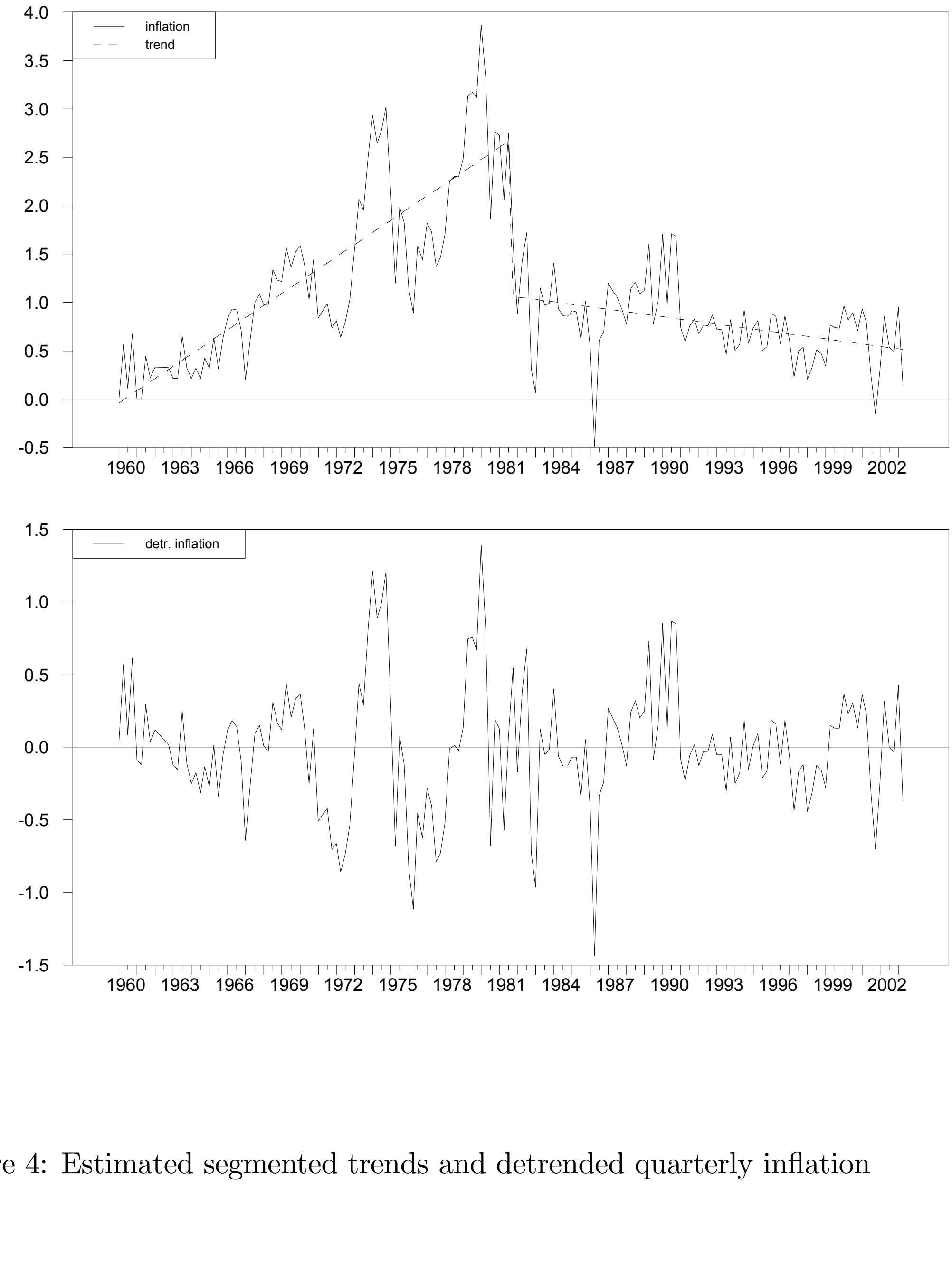

Figure 4: Estimated segmented trends and detrended quarterly inflation