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Asset pricing models with and without consumption data: An empirical evaluation

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Abstract

This paper evaluates the ability of the empirical model of asset pricing of Campbell (1993a,b) to explain the time-series and cross-sectional variation of expected returns of portfolios of stocks. In Campbell's model, an alternative risk-return relationship is derived by substituting consumption out of the linearized first-order condition of the representative agent. We compare this methodology to models that use actual consumption data, such as the model of Epstein and Zin, 1989, 1991, and the standard consumption-based CAPM. Although we find that Campbell's model fits the data slightly better than models which explicitly price consumption risk, and provides reasonable estimates of the representative agent's preference parameters, the parameter restrictions of the Campbell model, as well as its overidentifying orthogonality conditions, are generally rejected. The parameter restrictions of the Campbell model, and the overidentifying conditions, are marginally not rejected when the empirical model is augmented to account for the "size effect".

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1. Introduction

Based on Rubinstein (1976), Lucas (1978) and Breeden (1979), much of the recent work in equilibrium asset pricing seeks to link both the cross-sectional and the time-series pattern of expected asset returns with the pattern of covariances between realized returns and consumption growth. Empirical tests of a simple model of consumption and asset returns, the so-called Consumption Capital Asset Pricing Model (CCAPM), lead to strong statistical (Hansen and Singleton, 1983) and economic (Mehra and Prescott, 1985) rejections across a wide range of assets, and perform poorly when used to explain the cross-sectional variation in expected stock returns (Mankiw and Shapiro, 1986).

Researchers point out a number of shortcomings in the original tests of the CCAPM, however. First, the tests concentrate on a restrictive theoretical version of the model, which assumes that agents' preferences are of the time-separable, single parameter, isoelastic family. Second, the tests use consumption data, which are plagued by measurement error and time-aggregation bias (Grossman et al., 1987, Wheatley, 1988, and Breeden et al., 1989). Finally, the consumption of asset market participants may be poorly proxied by aggregate consumption (Mankiw and Zeldes, 1991).

Recently, alternative models which allow richer structures for agents' preferences (Weil, 1989, 1990, Epstein and Zin, 1989, 1991, Constantinides, 1990) have been introduced. Moreover, the aggregation and measurement problems in consumption have led researchers to seek alternative expressions for the consumption factor. Campbell (1993a,b) presents the most prominent effort of this kind. [1] By linearizing the representative consumer's budget constraint, Campbell expresses unanticipated consumption as a function of the expectational revisions in current and future returns on wealth. Using this expression to substitute consumption out of the representative agent model leads to a relation between an asset's expected return and the covariance of its return with the market return and with state variables that predict the sum of discounted future market returns. This approach results in a multi-factor asset pricing model similar to Merton's partial equilibrium model (Merton, 1973). In Campbell's model, however, priced risk factors are chosen using an explicit criterion: economic variables that predict the future return on wealth will be priced in equilibrium. This criterion supplements the economic intuition used in previous attempts to find macroeconomic factors, as in Chen et al. (1986) and deflects Fama's criticism (Fama, 1991) that the "measured relations between returns and economic factors are spurious" (Campbell, 1993b).

[1] Kazemi (1992) presents a model in which the rate of return on a very long-term default-free real consol bond is perfectly negatively correlated with the representative investor's marginal utility of consumption. Kazemi's result appears in the model of Campbell (1993a, p. 498) as well. Bossaerts and Green (1989) also derive a two-factor model in which real bonds play an important role in asset pricing. See also Breeden (1986).

In this paper we evaluate the empirical significance of generalized asset pricing models using Campbell's strategy of substituting measured consumption out of an empirical model of asset prices based on the work of Epstein and Zin and Weil. [2] Using regression analysis, we identify state variables which predict the return on the market portfolio. We then adopt the vector autoregression (VAR) approach of Campbell (1991, 1993a) to compute the conditional covariances of the state variables with returns on ten stock portfolios ranked annually by size. A consumption risk factor is constructed as a weighted average of the covariances of the state variables – the state variable risk factors – with weights derived from the VAR parameter estimates. The generalized method of moments (GMM) is used to estimate simultaneously the parameters of the VAR and the conditional form of consumption-based asset pricing models. We subsequently compare Campbell's empirical model with its unrestricted multi-factor version as well as with the empirical models that use actual consumption data. Other papers, most notably Campbell (1993b), have also attempted to apply Campbell's model to the data, but do not directly compare models with and without consumption data.

The paper is organized as follows: Section 2 reviews the non-expected utility model and the modifications made to substitute consumption out of the model. Section 3 describes the data, the choice of risk factors, and the VAR models that characterize the joint evolution of stock returns, the growth in consumption, and the remaining state variables. Section 4 presents empirical tests of the Campbell model and contrasts it with an unrestricted multi-factor model, the two-factor Epstein–Zin–Weil model using measured consumption data, and single factor models such as the static CAPM and consumption CAPM. Section 5 summarizes our findings.

2. Theoretical and econometric framework

2.1. Substituting consumption out of the model

Let C_t denote real per capita consumption at time t, W_t denote individual wealth at time t, $R_{m,t+1}$ denote the gross real rate of return on wealth (the "market portfolio"), and $E\{\cdot \mid \Omega_t\}$ represent the mathematical expectation conditional on the information at t. The representative consumer's objective in Epstein and Zin (1989, 1991) or Weil (1989, 1990) is to choose at each date investments

[2] In this regard, the work in this paper is similar to that of Campbell (1993b) and Li (1992) who also apply Campbell's model to the data. Campbell (1993b) departs from the standard practice in finance, however, by extending the empirical model to include human wealth as a component of total wealth. Unlike the current paper, Campbell (1993b) examines the unconditional version of his model. That is, he uses the unconditional moments of return innovations to estimate the model. In this regard Li's paper is closer in spirit to the current paper.

and a planned consumption path in order to maximize the recursively-defined function:

$$U_t = \left\{(1-\beta)C_t^{\frac{1-\gamma}{\theta}} + \beta\left(E\{U_{t+1} \mid \Omega_t\}\right)^{\frac{1}{\theta}}\right\}^{\theta}, \text{ s.t. } W_{t+1} = R_{m,t+1}(W_t - C_t). \tag{1}$$

In Eq. (1), β is the constant subjective discount factor, γ is the constant coefficient of relative risk aversion, and θ is equal to $(1-\gamma)/(1-1/\sigma)$, where σ is the elasticity of intertemporal substitution. When $\gamma = 1/\sigma$, and thus $\theta = 1$, Eq. (1) reduces to the objective function with time-separable power utility. For any asset i with a gross rate of return R_i, Epstein and Zin (1989, 1991) and Weil (1989, 1990) show that a maximizing consumer at an interior optimum will choose consumption such that the following Euler equation is satisfied:

$$E\left[\left\{\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\sigma}}\right\}^{\theta}\left\{\frac{1}{R_{m,t+1}}\right\}^{1-\theta} R_{i,t+1} \mid \Omega_t\right] = 1. \tag{2}$$

To empirically implement the above condition, Epstein and Zin (1991), Giovannini and Weil (1989), and Campbell (1993a) take a second-order Taylor approximation of Eq. (2) for any risky asset i as well as for a risk-free asset. (Alternatively, one can assume that the growth in consumption and the discrete asset returns are jointly log-normally distributed.) Denote the continuously compounded growth in real consumption by $\Delta c_{t+1} \equiv \ln(C_{t+1}/C_t)$, and the continuously compounded returns on aggregate wealth and any asset i by $r_{m,t+1} \equiv \ln(R_{m,t+1})$ and $r_{i,t+1} \equiv \ln(R_{i,t+1})$, respectively. The Taylor approximation results in the following risk-return relation:

$$E(r_{i,t+1} \mid z_t) - r_{f,t+1} = -\frac{1}{2}V_{ii,t} + (1-\theta)V_{im,t} + \frac{\theta}{\sigma}V_{ic,t}, \tag{3}$$

where $V_{ii,t} \equiv \text{Var}(r_{i,t+1} \mid z_t)$, $V_{ic} \equiv \text{Cov}(r_{i,t+1}, \Delta c_{t+1} \mid z_t)$, $V_{im} \equiv \text{Cov}(r_{i,t+1}, r_{m,t+1} \mid z_t)$, and z_t denotes a vector of predetermined variables that are elements of the information set Ω_t. When $\gamma = 1$ and thus $\theta = 0$, Eq. (3) collapses to a one-factor model like the logarithmic static CAPM. Alternatively, when $\gamma = 1/\sigma$ and thus $\theta = 1$, Eq. (3) collapses to the consumption CAPM. [3] Note that the coefficient of relative risk aversion, γ, is equal to the sum of the prices attached to market and consumption risk, that is: $(1-\theta) + \theta/\sigma = \gamma$, while the elasticity of

[3] When the elasticity of intertemporal substitution, σ, equals 1, θ becomes infinite, and the model resembles neither the static CAPM nor the consumption CAPM. In this case the consumption-wealth ratio (the marginal propensity to consume) is constant and $V_{im} = V_{ic}$. Giovannini and Weil (1989) show however, that asset pricing is not myopic unless γ is also unity.

intertemporal substitution, σ, is the ratio of one minus the coefficient on the market factor, $1-(1-\theta)$, divided by the coefficient on the consumption factor, θ/σ. If reliable consumption data exist, one can use Eq. (3) to assess the explanatory power of the model in the cross section and time series of asset returns. In the empirical section (Tables 4 and 5), we present the results of such an exercise. Our main interest, however, is in the form of the model that does not price consumption risk explicitly, and does not require the use of consumption data. We now review the derivation of this specification.

Linearizing the consumer's dynamic budget constraint around an average consumption-wealth ratio, letting c_t denote $\ln(C_t)$, and using the Euler Eq. (2), Campbell (1993a) derives an expression for the unanticipated component of current consumption:

$$\begin{aligned} c_{t+1} - E(c_{t+1} \mid z_t) &= r_{m,t+1} - E(r_{m,t+1} \mid z_t) + (1-\sigma)\left[E\left(\sum_{j=1}^{\infty} \rho^j r_{m,t+1+j} \mid z_{t+1}\right) \right. \\ &\qquad \left. - E\left(\sum_{j=1}^{\infty} \rho^j r_{m,t+1+j} \mid z_t\right)\right] \\ &\quad - \left[E\left(\sum_{j=1}^{\infty} \rho^j \mu_{m,t+1+j} \mid z_{t+1}\right) - E\left(\sum_{j=1}^{\infty} \rho^j \mu_{m,t+1+j} \mid z_t\right)\right]. \end{aligned} \tag{4}$$

In Eq. (4), $\mu_{m,t} \equiv \sigma \ \ln\beta + (1/2)(\theta/\sigma) \ \mathrm{Var}[\Delta c_{t+1} - \sigma \ \ r_{m,t+1} \mid z_t]$, and the discount factor ρ is equal to $1-\exp(a)$, where a is the mean log consumption-wealth ratio constructed from average sample values around the point of linearization. ρ thus reflects the *average* ratio of consumption to wealth. In the spirit of Cox et al. (1985) (CIR), Campbell imposes the condition:

$$\mu_{m,t} = \mu_0 + \psi E(r_{m,t+1} \mid z_t).$$

This is a sufficient but not necessary condition for substituting consumption out of the model. In the presence of time-varying conditional variance (heteroskedasticity) of consumption growth, we have $\psi \neq 0$. If the conditional variance of consumption growth relative to the return on wealth is constant (homoskedasticity), then $\psi = 0$. Making the appropriate substitution in Eq. (4) yields:

$$\begin{aligned} c_{t+1} - E(c_{t+1} \mid z_t) &= r_{m,t+1} - E(r_{m,t+1} \mid z_t) \\ &\quad + (1-\sigma-\psi)\left[E\left(\sum_{j=1}^{\infty} \rho^j r_{m,t+1+j} \mid z_{t+1}\right) - E\left(\sum_{j=1}^{\infty} \rho^j r_{m,t+1+j} \mid z_t\right)\right]. \end{aligned} \tag{5}$$

Substituting Eq. (5) in Eq. (3) results in an alternative form of Eq. (3) as follows:

$$E(r_{i,t+1} \mid z_t) - r_{f,t+1} = -\frac{1}{2} V_{ii,t} + \gamma V_{im,t} + (\gamma - 1)\left(1 + \frac{\psi}{\sigma - 1}\right) V_{ih,t}, \tag{6}$$

where $V_{ih,t}$ is the conditional covariance of asset i's return with the revision from t to $t+1$ in the expected discounted value of future returns on invested wealth:[4]

$$V_{ih,t} = \mathrm{Cov}\left[r_{i,t+1}, E\left(\sum_{j=1}^{\infty} \rho^j r_{m,t+1+j} \mid z_{t+1} \right) - E\left(\sum_{j=1}^{\infty} \rho^j r_{m,t+1+j} \mid z_t \right) \mid z_t \right].$$

Eq. (6) is a risk-return relationship that explicitly prices market risk and the risk attached to innovations in future investment opportunities. When $(\gamma - 1)[1 + \psi/(\sigma - 1)] > 0$, then an asset i whose returns are highly positively correlated with innovations in future returns on the market portfolio ($V_{ih} > > 0$) will have a higher expected return. This asset will be less desirable since it cannot be used to hedge against the risk of such innovations.

Although Eq. (6) does not *explicitly* price consumption risk, the average consumption-wealth ratio, reflected in the discount factor ρ, is embodied in the expression. Also, the derivation of Eq. (6) requires making a rather *ad hoc* assumption about the conditional second-order moment of consumption. Therefore, consumption is not completely endogenized in this model although the alternative assumptions concerning time variation in the second moments of consumption (i.e., either homoskedasticity or conditional heteroskedasticity of the CIR form) allow us to estimate the model without consumption data. The treatment of the time-varying covariances is a difficulty in Campbell's model as it ignores evidence that conditional heteroskedasticity for returns is typically of the ARCH form. Any rejection of the model under heteroskedasticity ($\psi \neq 0$), therefore, must be interpreted in light of a possible mis-specification of the form of the conditional heteroskedasticity.[5]

[4] It is easy to interpret the last term in Eq. 5 when $\sigma > 1$ or $\psi = 0$: When the coefficient of relative risk aversion γ is less than one, assets that have high returns whenever there is good news about future investment opportunities have lower mean returns. The intuitive explanation is that assets with positive covariances are desirable because they enable the consumer to profit from improved investment opportunities, but undesirable because they reduce the consumer's ability to hedge against a deterioration in such opportunities. Whenever $\gamma < 1$, that is, when investors do not care as much about the reduced hedging opportunities, the former effect dominates the latter; asset prices are higher and the mean return lower. When $\gamma = 1$, the two effects cancel each other out and Eq. (5) reduces to the traditional static CAPM where only the covariance with the market is relevant for asset pricing.

[5] Campbell (1993a,b) shows that when $\sigma = 1$ the two-factor model with time-varying covariances, $V_{im,t}$ and $V_{ic,t}$, holds exactly. Restoy (1992) linearizes the Euler Eq. (1) (instead of the budget constraint) and derives a similar two-factor model in which the time-varying covariances follow GARCH processes that are uncorrelated with the market.

In order to find an empirical proxy for $V_{ih,t}$, Campbell (1993a) proposes modeling the return on the market portfolio as the first element of the K-element state vector z_{t+1}, which is assumed to follow a first-order VAR [6] with a coefficient matrix A, and a disturbance vector $w = [w_{1,t}, \ldots, w_{k,t}]$:

$$z_{t+1} = Az_t + w_{t+1}. \tag{7}$$

The expected return on invested wealth in period $t+1+j$ conditional on z_t is written as:

$$E(r_{m,t+1+j} \mid z_t) = e1'A^{j+1}z_t,$$

where $e1$ is a conformable vector whose first element is unity and whose remaining elements are zero.

The discounted sum of revisions in forecasted r_m can now be written as:

$$E\left(\sum_{j=1}^{\infty} \rho^j r_{m,t+1+j} \mid z_{t+1}\right) - E\left(\sum_{j=1}^{\infty} \rho^j r_{m,t+1+j} \mid z_t\right)$$
$$= \lambda' w_{t+1}; \ \boldsymbol{\lambda} \equiv e1'\rho A(I - \rho A)^{-1}. \tag{8}$$

The vector $\boldsymbol{\lambda} = [\lambda_1, \ldots, \lambda_K]'$ has dimension equal to the number of state variables in the VAR. The covariance of asset i's return with the discounted sum of revisions in forecasted r_m can now be expressed as:

$$V_{ih,t} = \mathrm{Cov}[r_{i,t+1}, \lambda w_{t+1} \mid z_t] = \sum_{k=1}^{K} \lambda_k V_{ik,t}, \tag{9}$$

where $V_{ik,t} \equiv \mathrm{Cov}[r_{i,t+1}, w_{k,t+1} \mid z_t]$, $w_{k,t+1}$ being the kth element of the vector of VAR disturbances w_{t+1} and λ_k the kth element of vector $\boldsymbol{\lambda}$. Since by construction $V_{i1,t} \equiv V_{im,t}$, we can re-express Eq. (6) as:

$$E(r_{i,t+1} \mid z_t) - r_{f,t+1} = -\frac{1}{2}V_{ii,t} + \gamma V_{i1,t}$$
$$+ (\gamma - 1)\left(1 + \frac{\psi}{\sigma - 1}\right)\sum_{k=1}^{K} \lambda_k V_{ik,t}. \tag{10}$$

Eq. (10) has a form similar to a standard K-factor asset pricing model (Merton, 1973). In addition, Campbell's methodology suggests that the chosen factors ought to be related to variables that help predict market returns. The contribution of the VAR approach is a set of restrictions on the prices of these risk factors. Asset i's

[6] The assumption that z_t follows a *first*-order VAR is not restrictive because any p-order VAR system can be stacked into a companion-form first order system. For the sake of parsimony and computational ease, our empirical work deals exclusively with first-order VAR systems. We justify our use of a first-order system in Section 3.3.

covariance with the market, denoted by $V_{i1,t} \equiv V_{im,t}$ has a risk price of $\gamma + (\gamma - 1)[1 + \psi/(\sigma - 1)]\lambda_1$. The other factors have risk prices of $(\gamma - 1)[1 + \psi/(\sigma - 1)]\lambda_k$. That is, the risk price of each factor, k, where $k \neq 1$, is proportional to λ_k, where λ_k measures the expectational revision in the present value of market returns resulting from a unit innovation in the state variable k. This set of proportionality restrictions implies a *two-factor* model: the first factor is the market factor, V_{i1}, whose price is γ ; the second factor is the linear combination of covariance terms, $\sum \lambda_k V_{i,k}$, whose price is $(\gamma - 1)[1 + \psi/(\sigma - 1)]$.

A further restriction can be imposed if the conditional covariance of consumption growth relative to the return on wealth is constant, that is under homoskedasticity when $\psi = 0$. In this case, the two-factor Campbell model is expressed as follows:

$$E(r_{i,t+1} \mid z_t) - r_{f,t+1} = -\frac{1}{2}V_{ii} - \sum_{k=1}^{K} \lambda_k V_{ik} + \gamma\left(V_{i1} + \sum_{k=1}^{K} \lambda_k V_{ik}\right). \quad (11)$$

The above model identifies the coefficient of relative risk aversion γ and can be compared with the one-factor consumption CAPM, which also identifies γ.

2.2. Econometric issues

To simplify the estimation, we can eliminate the Jensen inequality term, $-(1/2)\ V_{ii,t}$, from Eq. (11) by substituting the expected discrete returns for the expected continuously compounded returns. To see this, note that if R_i denotes the discrete gross real rate of return on a risky asset or *portfolio* of assets i, R_f denotes the discrete gross real risk-free rate of return, and r_i and r_f denote their respective continuously compounded returns, then the following approximation holds: $E(r_{i,t+1} \mid z_t) + (1/2)\ V_{ii,t} - r_{f,t+1} \approx E(R_{i,t+1} \mid z_t) - R_{f,t+1}$. [7] We can thus write an approximate version of the risk-return relationship in Eq. (11) as:

$$E(R_{i,t+1} \mid z_t) - R_{f,t+1} \approx \gamma V_{i1,t} + (\gamma - 1)\left(1 + \frac{\psi}{\sigma - 1}\right)\sum_{k=1}^{K} \lambda_k V_{ik,t}. \quad (12)$$

Next, note that the conditional covariance between the return on asset i and state variable k, denoted by $V_{ik,t}$, is equal to $E[r_{i,t+1}\ w_{k,t+1}]$. That is, in order to form conditional covariances of portfolio returns with the state variables, it is not

[7] By definition, $R = \exp\{r\}$. Taking the second-order Taylor series expansion of $\exp\{r\}$ around $r = 0$ leads to: $R \cong 1 + r + (1/2)r^2$. It follows that $E_t R_{i,t+1} \cong 1 + E_t r_{i,t+1} + (1/2)\ [V_{ii,t} + (E_t r_{i,t+1})^2]$. Similarly, $R_{f,t+1} \cong 1 + r_{f,t+1} + (1/2)\ (r_{f,t+1})^2$. Assuming that the term $(1/2)$ $[(E_t r_{i,t+1})^2 - (r_{f,t+1})^2]$ is approximately zero, implies that $E_t R_{i,t+1} - R_{f,t+1} \cong E_t r_{i,t+1} - r_{f,t+1} + (1/2)\ V_{ii,t}$. Alternatively, under log-normality, the term $E(r_{i,t+1} \mid z_t) + (1/2)\ V_{ii,t}$ is exactly equal to $\ln(E(R_{i,t+1} \mid z_t))$. Approximating $\ln(E(R_{i,t+1} \mid z_t))$ by $E_t R_{i,t+1} - 1$ and $\ln(R_{f,t+1})$ by $R_{f,t+1} - 1$, leads to a similar expression as above.

necessary to compute portfolio return residuals based on regressions of the portfolio returns on the state variables. Hence, the number of estimable parameters is reduced considerably and the estimation becomes less cumbersome.

Our empirical models are based on the VAR in Eq. (8) and the *ex post* versions of the appropriate approximate risk-return relationships. [8] For example, the VAR and the approximation to Campbell's two-factor model under heteroskedasticity in Eq. (12) imply the empirical model:

$$\boldsymbol{w}_{t+1} = \boldsymbol{z}_{t+1} - \boldsymbol{A}\boldsymbol{z}_t;\ \lambda' \equiv e1'\rho\boldsymbol{A}(\boldsymbol{I} - \rho\boldsymbol{A})^{-1}$$

$$v_{i,t+1} = R_{i,t+1} - R_{f,t+1} - b_M(w_{1,t+1}r_{i,t+1}) - b_{SC}\sum_{k=1}^{K}\lambda_k(w_{k,t+1}r_{i,t+1});$$

$$i = 1,\dots,N, \tag{13}$$

where b_M is an estimate of γ and b_{SC} is the estimate of $(\gamma - 1)[1 + \psi/(\sigma - 1)]$. In Eq. (13), N denotes the number of assets/portfolios used to estimate the model, while $v_{i,t+1}$ denotes the innovation in the return on portfolio i relative to its conditional mean. The actual estimation allows for constant terms in the VAR, so that the state variables in z are interpreted as deviations from their respective means. Observe that in Campbell's two-factor model under heteroskedasticity ($\psi \neq 0$), the parameters ψ and σ cannot be separately identified. The empirical model corresponding to Campbell's two-factor model under homoskedasticity results when we restrict b_{SC} to equal $b_M - 1$ in Eq. (13).

Recall that the parameter ρ reflects the consumption-to-wealth ratio at the point of linearization. Because ρ is not a behavioral parameter we do not estimate it. Rather, in each of the empirical models that involve ρ, we set ρ equal to 0.985

[8] There is an important reason for the use of the approximate versions in our estimations, namely computability. Looking forward, the model in Table 5 (Panel A) of this paper estimates 14 equations with 34 estimable parameters under the approximate version. Using the exact version of the model, the number of equations would rise by 10 (one for each portfolio) to 24, and the number of estimable parameters would rise by 50 (10 equations × 5 instruments) to 84. Using GMM, estimations of such size rarely converge. To lend credence to the use of the approximation, we compared the results from an estimation of the exact model (when it did converge) to the results of the corresponding approximate model. This was possible for the case of the Epstein–Zin–Weil model of Table 5 (Panel D), for example. In the estimation of the exact version of this model, the estimated parameters and their standard errors are $b_M = 7.06$ (2.06) and $b_{\mathrm{GCON}} = 80.4$ (24.89), which are close to the corresponding estimates reported in Table 5. Alternatively, we can compare the reported results to the results of estimating a two-stage system of equations that treats the first-stage OLS residuals – from regressing each of the portfolio returns on the set of instruments – as data, but does not make use of the approximate model. The two stage procedure reduces the number of estimable parameters in the second-stage GMM estimation by 50. Although these subsystems are mis-specified, the two-stage estimation converges quickly because of the smaller number of estimable parameters in the second stage. The differences between the results reported in the tables, and those of the two-stage procedure, were again minor.

(equivalent to 0.941 at the annual frequency), effectively treating it as data.[9] This value corresponds to an average consumption-to-wealth ratio of six percent, which is the value used by Campbell (1993a,b) in his simulations. To test whether the results of our empirical work are sensitive to reasonably larger or smaller values of ρ, we also estimated each model using values of ρ corresponding to a two percent and an eight percent (annual) consumption-to-wealth ratio. We found that using the alternative values of ρ did not significantly affect the results or conclusions of our estimations.[10]

We estimate systems of equations such as Eq. (13) using the Generalized Method of Moments in case (ii) of Hansen (1982, p. 1043). Case (ii) allows for conditional heteroskedasticity of the error terms in $\boldsymbol{w}$ and the v_i.[11] In each version estimated, we use $K+1$ instruments: the K state variables of the VAR model plus a constant. The model above implies the orthogonality condition: $E[\boldsymbol{w}_{t+1}, v_{1,t+1}, \ldots, v_{N,t+1} \mid z_t, 1] = 0$. With $K+1$ instruments and N assets/portfolios ($i = 1, \ldots, N$), there are $[(K+1) \times (N+K)]$ orthogonality conditions. The number of over-identifying restrictions will be equal to the number of orthogonality conditions minus the number of estimated parameters. The number of estimated parameters will be determined by the particular specification of the model. To see how the estimation and the tests of the models are performed, let $\boldsymbol{\delta}$ denote the vector of the unknown (model plus VAR) parameters, and let $\boldsymbol{u}_{t+1} = [\boldsymbol{w}_{t+1}, v_{1,t+1}, \ldots, v_{N,t+1}]$ denote a vector of dimension $N+K$, which contains the K VAR residuals $w_{k,t+1}$ and the N asset/portfolio residuals $v_{i,t+1}$. Let also $f_t(\delta)$ denote the vector of $(N+K) \times (K+1)$ orthogonality conditions, $f_t(\delta) \equiv \mathrm{Vec}[\boldsymbol{u}_{t+1} \otimes (z_t, 1)]$. The parameter vector δ is chosen to make the orthogonality conditions as close to zero as possible by minimizing the quadratic $J_T(\delta)$, defined as follows:

$$J_T(\delta) = \boldsymbol{g}_T(\delta)' \boldsymbol{W}_T \boldsymbol{g}_T(\delta),$$

where

$$\boldsymbol{g}_T(\delta) = \frac{1}{T} \sum_{t=1}^{T} f_t(\delta); \ \boldsymbol{W}_T = \left(\frac{1}{T} \sum_{t=1}^{T} [\boldsymbol{u}_{t+1} \boldsymbol{u}'_{t+1} \otimes z_t z'_t] \right)^{-1}.$$

[9] Rather than impose a value, Li (1992) estimates ρ using the heteroskedastic model. Li's estimate of ρ is 0.919 (monthly) with a standard error of 0.041. This corresponds to a consumption-wealth ratio of 0.64 at an annual rate.

[10] More specifically, using the alternative values of ρ did not change the statistical inference derived from any tests of the orthogonality conditions or auxiliary parameter restrictions. Neither were any parameter estimates qualitatively different under the two alternative values of ρ.

[11] If the distribution of the equation error terms deviates from the joint normal distribution, then allowing for conditional heteroskedasticity leads to more robust tests of the various hypotheses. MacKinlay and Richardson (1991) show, for example, that traditional multi-variate Wald statistics of the hypothesis of mean-variance efficiency, which assume conditional homoskedasticity, lead to incorrect critical values; by contrast, statistics based on Hansen's case (ii) GMM provide robust tests.

The vector $g_T(\delta)$ has $(N+K)\times(K+1)$ elements, and the weighting matrix W_T has $(N+K)\times(K+1)$ rows and $(N+K)\times(K+1)$ columns. The minimization begins by setting W_T equal to the identity matrix, and the VAR parameters equal to their counterparts estimated by OLS. In each subsequent iteration, a new estimate of W_T is constructed from the estimated equation residuals of the previous step and the predetermined instruments, z_t. Hansen (1982) shows that the minimized value of J_T, denoted as J_Φ, is distributed asymptotically as a χ^2 statistic with degrees of freedom equal to the number of overidentifying restrictions conditions, namely, the total number of orthogonality conditions minus the number of estimated parameters. J_Φ provides a specification test of the model; a high J_Φ statistic indicates that the disturbance vector $[w'_{t+1}, v_{i,t+1}, \ldots, v_{N,t+1}]$ is correlated with the vector of instruments $[z_t, 1]$.

In the models of the later subsections, we are also interested in testing certain linear and non-linear parameter restrictions. For this purpose, we use a Wald statistic, constructed from the parameters of the unrestricted model. [12] Suppose, for example, we wish to test the two restrictions: $\xi_1(\delta)=\xi_2(\delta)=\xi_3(\delta)$ (as we do later in Table 5, Panel A), where $\xi_1(\delta)$, $\xi_2(\delta)$, and $\xi_3(\delta)$ are distinct scalar-valued non-linear functions of the model's vector of parameters δ. Define the vector $\xi \equiv (\xi_1, \xi_2, \xi_3)'$. The three by three variance-covariance matrix of ξ, Σ_ξ, equals $[\partial\xi/\partial\delta]'\Sigma[\partial\xi/\partial\delta]$, where Σ denotes the p by p variance-covariance matrix of δ, where p is the number of elements of vector δ, and $[\partial\xi/\partial\delta]$ is the p by three matrix of partial derivatives of the elements of vector ξ with respect to the elements of vector δ. Let P denote the two by three matrix of the above restrictions on vector ξ (the first row could be $(1,-1,0)$ and the second row $(1,0,-1)$). Then the Wald statistic,

$$\Phi_W \equiv [P\xi]'[P\Sigma_\xi P']^{-1}[P\xi],$$

is asymptotically a χ^2 statistic with two degrees of freedom.

Because our set of instruments corresponds to the set of state variables in the VAR, the subset of orthogonality conditions applied to the VAR errors, that is

[12] We also considered another statistic, suggested by Hansen. This statistic, call it Φ_H, resembles a likelihood ratio statistic and is computed as the difference in the J_Φ statistics from the restricted and unrestricted models. Hansen shows that under the restricted specification Φ_H is distributed asymptotically as a χ^2 statistic with degrees of freedom equal to the number of restrictions. Like the Wald statistic (Φ_W) we report, the Φ_H statistic will deviate from its asymptotic distribution in small samples. Moreover, in small samples, the weighting matrix, W_T (used to construct the J_H statistics), will differ between the restricted and unrestricted models. In cases where the parameter restrictions are not especially binding, different weighting matrices may cause the J_H statistic to be greater for the *unrestricted* specification than for the *restricted* specification, leading to a negative (i.e., perverse) Φ_H statistic. In contrast, the Wald statistic does not ever have this "problem", because its construction uses a single covariance matrix from the estimation of the unrestricted model only. For this reason, we report the Wald statistic, Φ_W. Although, as is well-known, the value of the likelihood ratio statistic is less than that of the Wald (so that use of the Wald favors rejection), in no cases did the use of Hansen's Φ_H statistic lead to statistical inferences that widely diverged from those using the Wald statistic Φ_H.

$E(w_{t+1} \mid z_t) = 0$, are the same orthogonality conditions that OLS imposes. Thus, if the system of equations in Eq. (13) consisted only of the K VAR equations, GMM and OLS would provide similar VAR parameter estimates. The additional portfolio equations, however, force the GMM estimates of the VAR parameters to differ from the OLS estimates. Such differences are likely to be larger when the estimated model does not properly account for the time-series and cross-sectional variation in the expected returns of the N assets/portfolios.

3. Data and the choice of risk factors

Our data are quarterly and run from 1959:1 through 1991:4. The beginning of the sample is dictated by the availability of consumption data, which begin in 1959 and are required in the comparison of models with and without consumption data. The quarterly frequency is the frequency also used by Mankiw and Shapiro (1986), who compare the CAPM with the consumption CAPM using consumption data. This frequency reduces the noise in consumption growth rates generated from the use of average – rather than month-end – consumption.

3.1. Constructing portfolios

The stock return data come from the monthly tapes of the Center for Research in Securities Prices (CRSP) and include all firms listed on the New York Stock Exchange (NYSE) and the American Stock Exchange (AMEX). We compute quarterly discrete individual firm returns by cumulating the consecutive monthly returns of a given quarter t as follows: $R_t - 1 \equiv (1 + y_{m,t})(1 + y_{m-1,t})(1 + y_{m-2,t}) - 1$, where $y_{m,t}$ denotes the discrete return over the last month of quarter t.

The method of constructing stock portfolios closely follows Fama and French (1992). To form portfolios, we pre-rank all firms in each year by the market value of their equity as of December of year $\tau - 1$, and subsequently allocate them into ten deciles in ascending order with an equal number of firms in each decile. Keeping the firms in the portfolios fixed, we then compute quarterly returns for the fiscal year that begins in July of year τ and ends in June of year $\tau + 1$. The portfolio returns are value-weighted, with weights proportional to the value of the firm at the end of the previous quarter. [13] The time series of portfolio returns can be viewed as the returns to mutual funds with changing compositions. [14]

[13] The gap of six months between the ranking date and the date we begin recording returns has no bearing on our analysis. It is chosen to facilitate comparisons with other work, as in Fama and French (1992), that uses accounting variables to rank firms into portfolios.

[14] Shanken and Weinstein (1990) use a similar approach to portfolio construction. They point out that other procedures which fix the firms in each portfolio according to a post-ranking characteristic (ranking, say, as of the end of a five year period that is used to estimate the betas) lead to biased beta estimates. Our pre-ranking procedure is immune to such biases.

Table 1 presents summary statistics on the portfolio quarterly returns. Returns are computed in two ways: Panel A presents continuously compounded real returns, corresponding to r_i, which serve as the basis for computing conditional covariances with the state variables. Panel B presents discrete excess returns, corresponding to R_i, which serve as the dependent variables in the later empirical analysis. The table shows that the smaller the size of the portfolio, the higher both its volatility and its average return. Observe that the excess skewness and excess kurtosis are statistically significant for many of the portfolios, suggesting non-trivial deviations from the normal distribution. Finally, as expected, the autocorrelations at lags one through four are close to zero.

3.2. Risk factors

Campbell's model suggests that priced factors should be found by choosing the variables that help forecast the return on wealth. Since this return is unobservable, we follow the practice of using the return of the aggregate stock market as a proxy for the return on wealth. Despite Roll's critique (Roll, 1977), Stambaugh (1982) provides some justification for this practice by showing that broader indices of wealth are highly correlated with the stock market index, apparently because the high volatility of the stock market index dominates the broader indices. An alternative approach is to introduce human capital directly in the measure of wealth (Campbell, 1993b). Although this is a valid approach in this context, we do not pursue it here. In our opinion, the measurement of human capital is potentially as fraught with error as the measurement of consumption.

In addition to the market return, a number of multi-factor studies choose macroeconomic variables as other potentially priced factors. This choice is based on economic intuition that may not be too different from the intuition that underlies the explicit criterion of stock return predictability. For example, both Chen et al. (1986) and Chan et al. (1985) include measures of changes in the quality spread and the term structure spread, and anticipated and unanticipated inflation. Ferson and Campbell (1991) include the change in the term structure spread, the real short-term rate, and growth in consumption. Most of these variables have been shown to predict stock returns, although the connection between a potentially priced factor and the predictability of stock returns is not made explicit.

The evidence on stock market forecastability is extensive (e.g., Campbell, 1987, Campbell and Shiller, 1988, Chen, 1991, Fama and French, 1989, Fama and Schwert, 1977, Froot, 1990, Hardouvelis and Wizman, 1992, Harvey, 1989, Hodrick, 1992 and Keim and Stambaugh, 1986). In this paper, we follow the common practice of using financial variables to predict stock returns. Financial variables immediately capture the beliefs and forecasts of market participants, who are forward looking. However, since the relation between financial variables and future stock returns is not structural, such a relation may deteriorate over time. For

Table 1

Summary statistics for decile portfolio real and excess returns, inflation, and risk-free returns [a]

Decile	Mean	Std. dev.	Excess skewness	Excess kurtosis	Autocorrelations			
					$\rho1$	$\rho2$	$\rho3$	$\rho4$
Continuously compounded quarterly real portfolio returns: $\ln(R_{i,t+1}) - \ln(\mathrm{CPI}_{t+1}/\mathrm{CPI}_t)$								
Smallest	0.020	0.165	0.06	0.71	−0.02	−0.06	−0.05	0.23
Second	0.017	0.150	−0.02	1.20 *	−0.03	−0.07	−0.07	0.20
Third	0.017	0.136	−0.13	0.90 *	−0.00	−0.04	−0.07	0.12
Fourth	0.017	0.128	−0.34	0.93 *	−0.01	−0.06	−0.06	0.07
Fifth	0.015	0.125	−0.38	1.16 *	−0.00	−0.07	−0.04	0.07
Sixth	0.018	0.120	−0.49 *	1.14 *	−0.04	−0.07	−0.04	0.03
Seventh	0.019	0.111	−0.59 *	1.19 *	0.02	−0.12	−0.05	0.01
Eighth	0.016	0.104	−0.62 *	0.97 *	0.03	−0.11	−0.07	−0.02
Ninth	0.016	0.097	−0.71 *	0.88 *	0.03	−0.14	−0.06	−0.04
Largest	0.011	0.082	−1.01 *	2.37 *	0.07	−0.11	−0.05	−0.01
Continuously compounded quarterly inflation and real risk-free returns								
$\pi_{t+1} = \ln(\mathrm{CPI}_{m,t+1}/\mathrm{CPI}_{m,t})$	0.12	0.009	0.89 *	0.62	0.74	0.70	0.70	0.59
$\ln(R_{f,t+1}) - \pi_{t+1}$	0.004	0.007	0.40	0.94 *	0.64	0.63	0.64	0.52
Discrete quarterly portfolio excess returns: $R_{i,t+1} - R_{f,t+1}$								
Smallest	0.030	0.174	0.72 *	1.38 *	−0.01	−0.09	−0.07	0.25
Second	0.024	0.157	0.70 *	1.84 *	−0.03	−0.08	−0.09	0.23
Third	0.023	0.141	0.46 *	1.17 *	−0.00	−0.05	−0.10	0.14
Fourth	0.022	0.130	0.21	0.97 *	−0.01	−0.08	−0.09	0.08
Fifth	0.018	0.126	0.20	1.33 *	−0.00	−0.09	−0.07	0.08
Sixth	0.022	0.121	0.02	0.83	−0.04	−0.09	−0.07	0.03
Seventh	0.022	0.111	−0.11	0.97 *	0.02	−0.14	−0.08	0.01
Eighth	0.017	0.103	−0.22	0.71	0.03	−0.13	−0.09	−0.02
Ninth	0.017	0.096	−0.36	0.57	0.03	−0.15	−0.08	−0.05
Largest	0.010	0.079	−0.66	1.32 *	0.06	−0.13	−0.08	−0.03
Risk-free return: $R_{f,t+1} - 1$								
	0.016	0.007	1.01 *	1.06 *	0.90	0.84	0.83	0.76

example, the spread between the excess return of a three-month T-bill over a one-month T-bill may be an excellent predictor for most of the sample, but financial innovations, or a shift in the focus of financial market participants away from the short-term actions of the Federal Reserve may alter the informative content of this variable. With this in mind, we are guided in our choice of risk factors by parsimony of the eventual model and the empirical robustness of the predictors as evidenced by similar regression coefficients in both univariate and multivariate regressions and across subperiods. On this basis the dividend yield and the first difference in the average quarterly three-month T-bill rate are chosen. Moreover, the dividend yield is the natural first candidate on theoretical grounds (Campbell and Shiller, 1988), while the T-bill yield has been shown to predict the excess returns on a variety of other assets (Froot, 1990). [15]

3.3. Summary statistics and OLS vector autoregressions

In addition to the dividend yield and the change in the three-month T-bill rate, the VAR also includes the real return on the aggregate stock market. This variable is not a good predictor of the future real stock returns, but we include it for completeness of the VAR. Although we first estimate the system without including consumption data (so that $K = 3$) we also use an augmented VAR model that does include consumption ($K = 4$). This is motivated by the desire to model consumption shocks in the same VAR framework, which is necessary because we will

[15] In a first-order VAR using a monthly sample, Hodrick (1992) uses the T-bill rate relative to a twelve-month moving average. Campbell and Ammer (1993) also de-trend the T-bill rate by a similar moving average. In our framework, the simple first difference of the T-bill yield provides similar predictions as the T-bill yield relative to a one-year backward moving average. We use the first difference in the T-bill rate to avoid including in the VAR a second highly persistent series in addition to the dividend yield series. The presence of two highly persistent series in a VAR may negate the validity of asymptotic distribution theory used to interpret test statistics (King et al. (1991)). The three-month T-bill rate relative to trend has slightly higher predictive power for stock returns than the simple first difference of the T-bill rate. The use of the T-bill rate relative to trend does not affect the GMM estimates of the model parameter but does affects the GMM estimates of the VAR; the own autoregressive coefficient of this state variable would occasionally exceed unity.

Notes to Table 1:

[a] The sample consists of 130 quarterly observations (1959:3–1991:4). All NYSE and AMEX firms in the CRSP tapes are pre-ranked by the market value of their equity as of December of year $\tau - 1$ and allocated into deciles ($i = 1,\dots,10$) in ascending order. Portfolio returns are then computed for the fiscal year that begins in July of year τ and ends in June of year $\tau + 1$. Quarterly gross portfolio returns are constructed from monthly returns as follows: $R_{i,t+1} = (1 + y_{m,t+1})(1 + y_{m,t+2})(1 + y_{m,t+3})$, where $y_{m,t+1}$ denotes the CRSP return of the last month of quarter t for portfolio i. Continuously compounded real returns are defined as $\ln(R_{i,t+1}) - \ln(\mathrm{CPI}_{t+1} / \mathrm{CPI}_t)$, where CPI_{t+1} is the consumer price index of the last month of quarter $t+1$. Excess discrete returns are defined as $R_{i,t+1} - R_{f,t+1}$, where $R_{f,t+1}$ is the three-month annualized bond-equivalent T-bill yield at the end of quarter t divided by four. * indicates significance at the 5% level.

Table 2
Summary statistics and VAR models of the state variables [a,b]

Panel A: Summary statistics

	Correlation matrix:								Autocorrelations:				Dickey–Fuller t-statistics
	RRET	DYLD	DTBL	GCON	Mean	STD	SKW	KUR	$\rho 1$	$\rho 2$	$\rho 3$	$\rho 4$	
RRET	1.00	−0.17	−0.25	0.26	0.13	0.087	−0.95 *	1.86 *	0.08	−0.14	−0.06	−0.03	−5.56 *
DYLD		1.00	−0.02	−0.23	0.038	0.009	0.64 *	−0.49	0.83 *	0.75 *	0.68 *	0.66 *	−2.34
DTBL			1.00	−0.11	0.000	0.010	−0.26	8.20 *	0.16	−0.27 *	0.11	0.06	−3.86 *
GCON				1.00	0.004	0.006	−0.16	0.29	0.23 *	0.15	0.19 *	0.07	−4.27 *

Panel B: First-order VAR model without consumption (three variables): $\boldsymbol{w}_{t+1} = z_{t+1} - Az_t$; $\boldsymbol{\lambda} \equiv e1'\rho A(I - \rho A)^{-1}$

Equation:	Coefficient values:				Residual statistics:			Autocorrelations:				Cumby–Huizinga statistic:		
	RRET	DYLD	DTBL	Rbar^2	STD	SKW	KUR	$\rho 1$	$\rho 2$	$\rho 3$	$\rho 4$	L4	L8	L12
RRET	0.06	2.14 *	−1.99 *	0.079	0.083	−1.16 *	1.98 *	−0.00	−0.13	0.01	−0.00	0.00	0.38	13.86
	(0.65)	(2.49)	(−2.46)									[1.00]	[1.00]	[0.31]
DYLD	−0.01	0.82 *	0.06	0.699	0.005	0.28	0.24	−0.28	0.09	−0.06	0.33	0.66	8.65	15.55
	(−1.62)	(16.7)	(1.37)									[0.96]	[0.37]	[0.21]
DTBL	0.02	−0.16	0.18	0.041	0.009	0.18	8.70 *	0.03	−0.30	0.19	0.06	1.01	1.82	2.92
	(1.20)	(−1.14)	(1.84)									[0.91]	[0.99]	[1.00]
λ	−0.06	11.52 *	−1.40	H0 of VAR parameter stability across 1959:3–75:4 and 1976:1–91:4: $\chi^2(12) = 18.6$ [0.10].										
	(−0.01)	(3.18)	(−1.47)											

Panel C: First-order VAR model with consumption (four variables): $\boldsymbol{w}_{t+1} = z_{t+1} - Az_t$; $\boldsymbol{\lambda} \equiv \equiv e1'\rho A(I - \rho A^{-1}$

Equation:	Coefficient values:					Residual statistics:			Autocorrelations:						
	RRET	DYLD	DTBL	GCON	Rbar^2	STD	SKW	KUR	$\rho 1$	$\rho 2$	$\rho 3$	$\rho 4$	L4	L8	L12
RRET	8w − 0.04	2.23 *	−2.07 *	0.80	0.074	0.083	−1.17 *	1.97 *	−0.01	−0.14	0.01	−0.01	0.00	0.60	7.05
	(0.65)	(2.49)	(−2.46)	(0.55)									[1.00]	[1.00]	[0.85]
DYLD	−0.01	0.81 *	0.07	−0.08	0.699	0.005	0.30	0.26	−0.27	0.07	−0.04	0.31	0.23	25.1 *	14.4
	(−1.28)	(16.2)	(1.52)	(−1.04)									[0.99]	[0.00]	[0.27]
DTBL	0.01	−0.12	0.15	0.30 *	0.060	0.009	0.43 *	8.69 *	0.05	−0.31	0.19	0.07	1.05	1.77	3.18
	(0.81)	(−0.96)	(1.48)	(2.28)									[0.90]	[0.99]	[0.99]
GCON	0.00	−0.10	−0.10	0.21 *	0.077	0.005	−0.06	0.12	−0.02	0.05	0.12	0.05	0.00	5.97	0.00
	(0.08)	(−1.77)	(−1.92)	(2.33)									[1.00]	[0.65]	[1.00]
λ	−0.04	11.56 *	−1.27	0.74		H0 of VAR parameter stability across 1959:3–75:4 and 1976:1–91:4: $\chi^2(20) = 25.3$ [0.19]									
	(−0.00)	(3.13)	(−0.27)	(−0.45)											

subsequently be interested in comparing Campbell's model with intertemporal asset pricing models that do use consumption data.

The VAR state variables are denoted as follows: RRET is the continuously compounded quarterly real return on the value-weighted NYSE index, constructed from the CRSP monthly total returns and the Bureau of Labor Statistics consumer price index. DYLD is the quarterly dividend yield on the NYSE index of stocks. DTBL is the change in the average quarterly three-month T-bill yield. Finally, GCON represents the quarterly growth in per-capita consumption from the last month of the previous quarter to the last month of the current quarter. More detailed definitions of the VAR state variables are found in the footnote of Table 2.

Panel A of Table 2 provides summary statistics on the VAR variables. The contemporaneous correlations between the four variables are low, suggesting that each one contains independent information. The Dickey–Fuller tests in the last column reject the hypothesis of a unit root in all state variables but the dividend yield. We attribute the lack of rejection in the case of the dividend yield to the lack of statistical power arising from the size of the sample. Previous investigators reject the unit root hypothesis for the dividend yield using the sample from 1926 to the present, but fail to reject it using post-war samples (see Schwert, 1987, Campbell and Shiller, 1988).

Notes to Table 2:

[a] The sample consists of 130 quarterly observations (1959:3–1991:4). STD, SKW, and KUR denote standard deviation, excess skewness, and excess kurtosis, respectively. In Panel A, the Dickey–Fuller t-statistic tests the null hypothesis of non-stationarity. The null hypothesis is expressed as $\alpha = 0$ in the regression equation: $\Delta Y_t = \beta_0 + \alpha Y_{t-1} + \beta_1 \Delta Yt - 1 + \ldots + \beta_4 Y_{t-4} + u_t$. In Panels B and C, λ denotes the factor weights of the state variables implied by the parameters of the VAR; and L4, L8, and L12, are the Cumby and Huizinga (1992) χ^2 statistics of the hypothesis that the first, four, eight, and twelve VAR residual autocorrelations are zero. Heteroskedasticity-consistent t-statistics are inside the parentheses, and probability values levels inside the brackets. * indicates significance at the 5% level.

[b] The state variables are defined as follows:

RRET$_t$: Continuously compounded value-weighted quarterly NYSE real return: $\ln[(1+y_{m,t})(1+y_{m-1,t})(1+y_{m-2,t})] - \ln(\text{CPI}_{m,t}/\text{CPI}_{m-1,t})$, where $y_{m,t}$ denotes the discrete return of the last month of quarter t (source: CRSP data tapes) and $\text{CPI}_{m,t}$ denotes the monthly consumer price index of the last month of quarter t (source: Bureau of Labor Statistics).

DYLD$_t$: Quarterly dividend yield on the NYSE index constructed from monthly dividends and the end-of-quarter NYSE index: $(D_{m,t} + D_{m-1,t} + D_{m-2,t})/P_t$. The monthly dividends are constructed from two CRSP series of monthly NYSE returns, one hat includes dividend payments, y_m, and a second return series that does not, $y_{x,m}$. Setting the base period price to 100, a monthly price series is derived by solving forward the equation: $P_m = P_{m-1}(1 + y_{x,m})$. The dividend of month m is subsequently defined as follows: $D_m = P_{m-1}(y_m - y_{x,m})$.

DTBL$_t$: First difference in the average quarterly bond-equivalent three-month T-bill yield (source: CITIBASE).

GCON$_t$: Quarterly growth in per-capital real monthly consumption: $\ln(C_{m,t}/C_{m,t-1})$, where $C_{m,t}$ equals real non-durable plus services purchases (1982 dollars) divided by population age 16 and over in the last month of quarter t.

Panel B of Table 2 presents the results of estimating by OLS a first-order vector autoregression using the three state variables: RRET, DYLD, and DTBL. The corresponding estimates of the vector λ, based on the OLS estimates of the VAR, are presented as benchmarks against which we can later compare the GMM estimates of λ. The VAR has stable coefficients: the hypothesis of VAR parameter stability cannot be rejected at conventional levels when the sample is partitioned in the middle. The first row of the panel describes the results of the RRET equation. Observe that a high dividend yield this quarter forecasts a high real stock return next quarter. This relationship is intuitive. If the high dividend yield is caused by a drop in the stock price in response to an increase in the risk premium, the subsequent rise in stock returns represents the anticipated reward for the extra risk investors undertook. Observe also that a low T-bill yield leads to a high subsequent real stock return. This correlation could also be due to a risk premium. An increase in risk, which pushes stock prices down, pushes T-bill prices up and their yield down as investors fly to quality. [16]

The remaining two equations in Panel B of Table 2 describe the behavior of the dividend yield, DYLD, and the change in the T-bill rate, DTBL. DYLD has a strong autoregressive component, but the DTBL is much harder to forecast. The autocorrelations of the estimated residuals in both equations give the impression of some extra dynamics that the first-order VAR is unable to capture. However, in both equations the sum of these autocorrelations is close to zero, suggesting that the cumulative long-run impact of shocks to the three variables is approximately the same whether one uses a first-order VAR framework or a higher-order one. Moreover, Cumby and Huizinga (1992) tests fail to reject the null hypothesis that the first four (L4), eight (L8), or twelve (L12) residual autocorrelations are jointly zero. [17] For computational simplicity, the models we estimate in Section 3 utilize a first-order VAR model. Hodrick (1992), who uses the dividend yield and the relative T-bill in a monthly sample, also adopts a first-order autoregression. [18]

The last row in Panel B of Table 2 calculates the multipliers λ_k of Eq. (6). Recall that each λ_k reflects the sensitivity of the discounted present value of returns to invested wealth to a unit shock in each state variable. Each element of the vector λ is a non-linear function of the parameters of the VAR, $\lambda = f(\alpha)$,

[16] Panel A shows that the correlation between DYLD and DTBL is zero, which suggests that DTBL captures a different type of risk from the one captured by DYLD. Indeed, in the presence of DYLD, DTBL continues to have significant marginal explanatory power for future stock returns.

[17] The tests of Cumby and Huizinga do not treat the residuals as data, but allow for the fact that these residuals contain sampling error. In the consumption equation residuals of Panel C, the L12 Cumby–Huizinga statistic resulted in a negative-definite covariance matrix. We also constructed the usual Box–Pierce tests (Box and Pierce, 1970) for the same set of autocorrelations and, for the most part, these tests fail to reject as well.

[18] Moreover, Hodrick's simulations show that a parsimonious VAR model used to assess long-run forecastability provides more reliable test statistics than multi-period regression models.

where α denotes a vector containing the VAR parameters. The t-statistics in parentheses are based on asymptotic standard errors, calculated from the diagonal elements of the matrix $[\partial f/\partial\alpha]' \ V \ [\partial f/\partial\alpha]$, where V is the variance-covariance matrix of the estimated VAR parameters α, and $[\partial f/\partial\alpha]$ denotes the matrix of derivatives of the elements of the vector λ with respect to each element of the vector α. Observe that the multiplier of the dividend yield shock, λ_{DYLD}, is positive, large, and statistically significant. This evidence is consistent with the theoretical importance of the dividend yield as a predictor of future stock returns. The multiplier of a shock to the T-bill yield, λ_{DTBL}, is negative with a t-statistic of -1.47. The multiplier of a shock to real aggregate stock returns, λ_{RRET}, is almost zero.

Panel C of Table 2 augments the three-variable VAR of Panel B by adding consumption growth, GCON. Consumption growth has very little marginal explanatory power in the stock return or the dividend yield regressions, but does help predict the subsequent change in the T-bill. Predicting consumption growth is as difficult as predicting real stock returns; the adjusted R^2 of the consumption equation is only 7.7 percent, about the same as in the stock return equation. Observe also that the vector of multipliers λ of the three original variables remains approximately the same. The multiplier of consumption growth, λ_{GCON}, is negative but insignificantly different from zero.

4. Asset pricing models: The evidence

4.1. Models that do not use consumption data

We begin in Table 3 by estimating models that do not make use of measured consumption data, neither as an explicitly priced risk factor nor as a state variable or instrument. This provides a benchmark against which we can compare empirical models that use measured consumption. We use a first-order VAR based on the first three ($K = 3$) state variables ($k =$ RRET, DYLD, DTBL). All models are estimated using four ($K + 1$) instruments (a constant and the lagged values of RRET, DYLD, and DTBL). This results in $52 = [(10 + 3) \times (3 + 1)]$ orthogonality conditions in each of the models estimated in Table 3.

Panel A of Table 3 presents estimates from an unrestricted three-factor asset pricing model. The model is the empirical analog of Eq. (12) without the restrictions on the factor prices that Campbell's model imposes. The model's J_{Φ} statistic rejects the overidentifying orthogonality conditions (p-value $= 0.03$). The estimated factor weight λ_{DYLD} is positive and significant, suggesting that positive shocks to the dividend yield are associated with higher expectations of future returns. A portfolio with a high covariance with DYLD, therefore, provides a poor hedge against a deterioration of future growth opportunities. Assuming that $(\gamma - 1)(1 + \psi/(\sigma - 1)) > 0$, this means that the price associated with dividend

Table 3

Intertemporal asset pricing models: GMM estimates from a three-variable VAR and ten size-portfolio equations [a]

Panel A: Unrestricted three-factor model

$v_{i,t+1} = R_{i,t+1} - R_{f,t+1} - b_{\mathrm{RRET}}(w_{\mathrm{RRET},t+1} r_{i,t+1}) - b_{\mathrm{DYLD}}(w_{\mathrm{DYLD},t+1} r_{i,t+1}) - b_{\mathrm{DTBL}}(w_{\mathrm{DTBL},t+1} r_{i,t+1})$; $i = 1,\ldots,10$

Parameter estimates and t-statistics						Model tests and statistics	
b_{RRET}	b_{DYLD}	b_{DTBL}	λ_{RRET}	λ_{DYLD}	λ_{DTBL}	$J_{\Phi}(37)$	H0: $\lambda_{\mathrm{RRET}} / b_{\mathrm{RRET}} = \lambda_{\mathrm{DYLD}} / b_{\mathrm{DYLD}} =$
5.83 *	191.3 *	164.7 *	−0.08	8.89 *	−2.63 *	54.6	$\lambda_{\mathrm{DTBL}} / b_{\mathrm{DTBL}}$;
(2.87)	(4.10)	(6.35)	(−0.02)	(5.42)	(−3.31)	[0.03]	$\Phi_W(2) = 13.3$ [0.00]

Panel B: Campbell two-factor model under heteroskedasticity ($\psi \neq 0$)

$v_{i,t+1} = R_{i,t+1} - R_{f,t+1} - b_M(w_{1,t+1} r_{i,t+1}) - b_{SC}\Sigma_{k=1}^{K}(\lambda_k w_{k,t+1} r_{i,t+1})$; $i = 1,\ldots,10$

Parameter estimates and t-statistics					Model tests and statistics	
b_M	b_{SC}	λ_{RRET}	λ_{DYLD}	λ_{DTBL}	$J_{\Phi}(38)$	H0: $b_{SC} = b_M - 1$;
8.61 *	29.2 *	−0.01	10.88 *	−0.43	52.3	$\Phi_W(1) = 25.3$ [0.00]
(3.65)	(5.26)	(−0.00)	(5.32)	(−0.53)	[0.06]	

Panel C: Campbell one-factor model

$v_{i,t+1} = R_{i,t+1} - R_{f,t+1} - b_M(w_{1,t+1} r_{i,t+1}) - (b_M - 1)\Sigma_{k=1}^{K}(\lambda_k w_{k,t+1} r_{i,t+1})$; $i = 1,\ldots,10$

Parameter estimates and t-statistics				Model tests and statistics
b_M	λ_{RRET}	λ_{DYLD}	λ_{DTBL}	$J_{\Phi}(39)$
1.44	0.01	−1.15	0.13	57.6
(1.40)	(0.06)	(−1.05)	(0.90)	[0.07]

[a] Results are based on 130 observations (1959:3–1991:4). The excess return on portfolio i, $R_{i,t+1} - R_{f,t+1}$, is calculated as the discrete return from the end of quarter t to the end of quarter $t+1$ minus the annualized three-month bond-equivalent T-bill yield at the end of quarter t divided by four. In Panels A, B, and C, the ten portfolio equations are estimated jointly with the three-variable VAR (Table 2, Panel B) using the generalized method of moments and allowing for conditional heteroskedasticity (case (ii) of Hansen (1982), p. 1043). The GMM instruments are: constant, RRET_t, DYLD_t, and DBLT_t, resulting in $4 \times (10+3) = 52$ orthogonality conditions. * indicates significance at the 5% level and t-statistics are reported in parentheses under the parameter estimates. J_{Φ} (d.f.) is the Hansen (1982) χ^2-statistic with degrees of freedom equal to the number of orthogonality conditions minus the number of estimated parameters. Φ_W is a Wald χ^2-statistic associated with the specified parameter restrictions and is based on the unrestricted model. Probability values for J_{Φ} (d.f.) and Φ_W (d.f.) are reported in square brackets under the model test statistics.

yield risk, b_{DYLD}, should be positive. This is, indeed, the case in Panel A of Table 3. On the other hand, Panel A shows that b_{DTBL} is also positive and significant, despite that λ_{DTBL} is negative. For this reason, the test of the restrictions of Campbell's model – that is, $\lambda_{\mathrm{DYLD}}/b_{\mathrm{DYLD}} = \lambda_{\mathrm{DTBL}}/b_{\mathrm{DTBL}}$ – is strongly rejected by the Wald statistic (p-value = 0.00).

Despite the rejection of Campbell's restrictions, Panel B of Table 3 estimates the Campbell two-factor model under heteroskedasticity ($\psi \neq 0$) by imposing the restrictions of the model directly. Note that Hansen's, J_{Φ}, statistic marginally rejects the two-factor model's overidentifying restrictions (p-value = 0.06). The two-factor model interprets the coefficient of b_M in Panel B as the parameter of relative risk aversion γ. This estimated coefficient is 8.61 (standard error = 2.36), and is statistically significant. The model does not allow us, however, to separately identify the estimate of the elasticity of intertemporal substitution, σ.

The estimated size of the coefficient of relative risk aversion, γ, in Panel B is of the same order of magnitude as the estimates provided by Campbell (1993b). Campbell uses two alternative samples, a monthly one from 1952 to 1990, and an annual one from 1871 to 1990. His monthly data set is richer than ours, comprising a cross-section of 22 stock and 3 bond portfolios, but his estimates are based on an unconditional specification of the model, making his estimation problem more manageable. While his model includes proxies for human capital, he also presents comparable estimates of γ under the assumption that the contribution of human capital to the wealth portfolio is zero. In Campbell's monthly and annual samples the estimates of γ are 15.6 and 2.7, respectively.

Within the context of the consumption CAPM, Mehra and Prescott (1985) claim that the coefficient of relative risk aversion that explains the average historical return on the stock market is implausibly high. Black (1990) and Kandel and Stambaugh (1991) subsequently point out that the belief (arising from static CAPM estimates as in Friend and Blume, 1975) that the true coefficient should be lower may be erroneous. Traditional estimates assumed, for example, that stock returns are i.i.d., when in reality stock returns may exhibit mean reversion. Mean reversion reduces the riskiness of the stock market and, within the context of the static CAPM, results in larger estimates of the coefficient of relative risk aversion. Kandel and Stambaugh show that typical values of γ ought to be about 30, rather than 3 (as Friend and Blume suggest). Note that in both our specification and Campbell's, V_{ih} reflects temporal dependence in stock returns and, hence, our estimate of γ is immune to the criticism of Black and Kandel and Stambaugh.

Panel B of Table 3 also tests the hypothesis that the Campbell model collapses to a one-factor model. This is equivalent to the restriction that $b_{SC} = b_M - 1$. This restriction is tested in Panel B using the Wald statistic and is strongly rejected (p-value = 0.00). This rejection constitutes perhaps indirect evidence that the conditional covariances of the model are time-varying and, hence, justifies our use of GMM. For completeness, in Panel C, we impose the restriction $b_{SC} = b_M - 1$ directly and estimate Campbell's one-factor model, which obtains when $\sigma = 1$.

Table 4

Intertemporal asset pricing models: GMM estimates from a four-variable VAR and ten size-portfolio equations [a]

Panel A: Unrestricted four-factor model

$v_{i,t+1} = R_{i,t+1} - R_{f,t+1} - b_{RRET}(w_{RRET,t+1} r_{i,t+1}) - b_{DYLD}(w_{DYLD,t+1} r_{i,t+1}) - b_{DTBL}(w_{DTBL,t+1} r_{i,t+1}) - b_{GCON}(w_{GCON,t+1} r_{i,t+1});\quad i = 1,\dots,10$

Parameter estimates and t-statistics								Model tests and statistics	
b_{RRET}	b_{DYLD}	b_{DTBL}	b_{GCON}	λ_{RRET}	λ_{DYLD}	λ_{DTBL}	λ_{GCON}	$J_{\Phi}(46)$	H0: $\lambda_{RRET}/b_{RRET} = \lambda_{DYLD}/b_{DYLD} =$
4.57	181.9 *	161.8 *	−0.39	−0.04	8.99 *	−3.12 *	0.63	57.8	λ_{GCON}/b_{GCON};
(1.90)	(3.48)	(7.16)	(−0.01)	(−0.01)	(6.50)	(−3.83)	(0.61)	[0.11]	$\Phi_W(2) = 15.3$ [0.00]

Panel B: Campbell two-factor model under heteroskedasticity ($\psi \neq 0$)

$v_{i,t+1} = R_{i,t+1} - R_{f,t+1} - b_M(w_{1,t+1} r_{i,t+1}) - b_{SC}\Sigma_{k=1}^{K}(\lambda_k w_{k,t+1} r_{i,t+1});\quad i = 1,\dots,10$

Parameter estimates and t-statistics						Model tests and statistics	
b_M	b_{SC}	λ_{RRET}	λ_{DYLD}	λ_{DTBL}	λ_{GCON}	$J_{\Phi}(48)$	H0: $b_{SC} = b_M - 1$;
4.72 *	19.26 *	0.02	12.47 *	0.45	1.62 *	58.12	$\Phi_W(1) = 26.1$ [0.00]
(2.58)	(3.95)	(0.00)	(4.44)	(0.51)	(2.01)	[0.15]	

Panel C: Campbell one-factor model

$v_{i,t+1} = R_{i,t+1} - R_{f,t+1} - b_M(w_{1,t+1} r_{i,t+1}) - (b_M - 1)\Sigma_{k=1}^{K}(\lambda_k w_{k,t+1} r_{i,t+1});\quad i = 1,\dots,10$

Parameter estimates and t-statistics					Model tests and statistics
b_M	λ_{RRET}	λ_{DYLD}	λ_{DTBL}	λ_{GCON}	$J_{\Phi}(49)$
1.60	0.03	−2.28	0.24	−0.10	64.25
(1.61)	(0.05)	(−1.67)	(0.99)	(−0.22)	[0.07]

Panel D: Epstein–Zin–Weil model

$v_{i,t+1} = R_{i,t+1} - R_{f,t+1} - b_M(w_{RRET,t+1} r_{i,t+1}) - b_{GCON}(w_{GCON,t+1} r_{i,t+1});\quad i = 1,\dots,10$

Parameter estimates and t-statistics		Model tests and statistics	
b_M	b_{GCON}	$J_{\Phi}(48)$	H0: $b_M + b_{GCON} = 1$
−8.55 *	284.9 *	70.3	(logarithmic utility);
(−5.96)	(11.3)	0.02	$\Phi_W(1) = 117.9$ [0.00]

Panel E: Campbell two-factor model augmented with consumption factor

$v_{i,t+1} = R_{i,t+1} - R_{f,t+1} - b_M(w_{1,t+1} r_{i,t+1}) - b_{SC}\Sigma_{k=1}^{K}(\lambda_k w_{k,t+1} r_{i,t+1}) - b_{GCON}(w_{GCON,t+1} R_{i,t+1})$; $i = 1,\ldots,10$

Parameter estimates and t-statistics							Model tests and statistics
b_M	b_{SC}	b_{GCON}	λ_{RRET}	λ_{DYLD}	λ_{DTBL}	λ_{GCON}	$J_\Phi(47)$
4.04	17.43 *	13.03	0.018	12.69 *	0.49	1.40	59.4
(1.89)	(3.95)	(0.40)	(0.00)	(4.61)	(0.53)	(1.67)	[0.11]

Panel F: Static CAPM and consumption CAPM

$v_{i,t+1} = R_{i,t+1} - R_{f,t+1} - b_k(w_{k,t+1} r_{i,t+1})$; $k = \text{RRET}, \text{GCON}$; $i = 1,\ldots,10$

Parameter estimates and t-statistics		Model tests and statistics
b_M	b_{GCON}	$J_\Phi(49)$
4.79 *	-	70.4
(5.82)		[0.02]
-	122.4 *	69.3
	(10.2)	[0.03]

[a] Results are based on 130 observations (1959:3–1991:4). The excess return on portfolio i, $R_{i,t+1} - R_{f,t+1}$, is calculated as the discrete return from the end of quarter t to the end of quarter $t+1$ minus the annualized three-month bond-equivalent T-bill yield at the end of quarter t divided by four. In Panels A, B, C, D, E, and F, the ten portfolio equations are estimated jointly with the four-variable VAR (Table 2, Panel C) using the generalized method of moments and allowing for conditional heteroskedasticity (case (ii) of Hansen (1982), p. 1043). The GMM instruments are: constant, RRET_t, $DYLD_t$, $DBLT_t$, and $GCON_t$, resulting in $5\times(10+4) = 70$ orthogonality conditions. * indicates significance at the 5% level and t-statistics are reported in parentheses under the parameter estimates. J_Φ (d.f.) is Hansen's (1982) χ^2-statistic with degrees of freedom equal to the number of orthogonality conditions minus the number of estimated parameters. Φ_W is a Wald χ^2-statistic associated with the specified parameter restrictions and is based on the unrestricted model. Probability values for J_Φ (d.f.) and Φ_W (d.f.) are reported in square brackets under the model test statistics.

Now, the estimate of the coefficient of risk aversion (γ) is given by $b_M = 1.44$ (standard error = 1.03), and is no longer significant. Also, the vector $\boldsymbol{\lambda}$ implied by the VAR parameter estimates deviates substantially from its OLS counterpart presented earlier in Table 2.

4.2. Models that use consumption data

In order to compare Campbell's model with other empirical asset pricing models, we are required to construct conditional covariances of portfolio real returns with consumption growth. To construct conditional covariances of consumption growth with asset returns, we could estimate a model of consumption growth unrelated to the earlier three-variable VAR. However, enlarging the earlier VAR model to include consumption growth facilitates our intended comparison because it provides a general four-factor framework that nests all relevant models. Note also that excluding measured consumption from the list of explicitly priced factors of the Campbell model while including it as a state variable in the VAR does not violate the spirit of that model. Although the observed consumption data series may not represent true consumption in Campbell's framework, it may still be used as a state variable that predicts the present value of future investment opportunities. The significance of consumption in helping to predict the T-bill yield in the VAR estimated in Panel C of Table 2 justifies this approach in part.

In Table 4, we estimate various asset models using the four-variable (RRET, DYLD, DTBL, and GCON), first-order VAR system. We use the lagged values of the same variables plus a constant as the set of instruments in the GMM procedure. This set of instruments results in 70 ($= [10 + 4] \times 5$) orthogonality conditions. In Panel A, we estimate an unrestricted four-factor model, including measured consumption growth as an explicitly priced factor. In this model, the J_{Φ} statistic marginally fails to reject the model's overidentifying restrictions (p-value = 0.11). As in Panel A of Table 3, however, b_{DYLD} and b_{DTBL} share the same sign and are statistically significant, while λ_{DYLD} and λ_{DTBL} are of opposite sign. This suggests a rejection of the restrictions of the Campbell model – $\lambda_{\mathrm{DYLD}}/b_{\mathrm{DYLD}} = \lambda_{\mathrm{DTBL}}/b_{\mathrm{DTBL}} = \lambda_{\mathrm{GCON}}/b_{\mathrm{GCON}}$ – which is confirmed by the Wald statistic ($\Phi_W(2) = 15.3$; p-value = 0.00) in Panel A. Interestingly, measured consumption risk is not significantly priced in the four-factor model.

In Panel B, we again estimate Campbell's two-factor model under heteroskedasticity ($\psi \neq 0$). Both factor prices b_M and b_{SC} are positive and significant, yielding a value of 4.72 (standard error = 1.83) for the coefficient of relative risk aversion. Unlike in Table 3, the overidentifying restrictions of the model are not rejected (p-value = 0.15). The test of the restriction of the Campbell two-factor model under homoskedasticity ($\psi = 0$) – that is $b_{SC} = b_M - 1$ – again rejects strongly (p-value = 0.00). Nevertheless, Panel C imposes the restriction $b_{SC} = b_M - 1$ and finds a coefficient of relative risk aversion equal to 1.60, which is statistically insignificant. Moreover, the J_{Φ} statistic marginally rejects the overidentifying restrictions of the model (p-value = 0.07).

Treating measured consumption as an explicitly priced factor, Panel D presents the results of estimates from the (approximate) Epstein–Zin–Weil two-factor model. The Epstein–Zin–Weil two-factor model does not perform as well as Campbell's two-factor model on both statistical and economic grounds. Unlike Campbell's two-factor model in Panel B, the J_{Φ} statistic strongly rejects the overidentifying restrictions of the Epstein–Zin–Weil model (p-value = 0.02). However, both factors are statistically significant. [19] The parameter estimates in Panel D can be used to uncover estimates of γ and σ. From Eq. (3), recall that $\text{plim}(b_M) = 1 - \theta$ and $\text{plim}(b_{SC}) = \theta/\sigma$, where $\theta \equiv (1-\gamma)/(1-1/\sigma)$. It follows that the implied estimate of σ, the elasticity of intertemporal substitution is roughly 0.03 (s.d. = 0.03) and not significantly different than zero. The implied estimate of the coefficient of risk aversion, γ, is 276.3 ($= b_M + b_{SC}$) with an associated standard error of 25.42.

In Panel E we estimate a general model which nests the Campbell two-factor model under heteroskedasticity and the Epstein–Zin–Weil model as special cases. The model effectively augments the Campbell model with the measured consumption factor, resulting in an empirical model that is not the consequence of any theoretical model. Note that in the presence of the Campbell model, the coefficient on the measured consumption factor is not significant ($b_{\text{GCON}} = 13.03$, s.d. = 32.58) while the risk factors in the Campbell model continue to be priced marginally significantly.

Panel F estimates two empirical models based on the consumption CAPM and the static CAPM, respectively. The consumption CAPM can be thought of as a restriction on the two-factor Epstein–Zin–Weil model where $\gamma = 1/\sigma$. The estimate of the parameter of relative risk aversion is 122.4 (s.d. = 12.0), and is highly significant. Given Mehra and Prescott's equity premium puzzle (Mehra and Prescott, 1985), the finding that the estimate of γ here and in Panel D is large may not come as a surprise. The estimate of γ is several standard deviations larger than the value that some authors (e.g., Kandel and Stambaugh, 1991) suggest is reasonable. Apparently, when estimating consumption-based asset pricing models using actual consumption data, the low intertemporal variability in the consumption growth requires a larger risk aversion parameter to explain the cross-sectional and time-series variation in risk premia. [20] The larger estimate of the coefficient of relative risk aversion, γ, in the consumption-CAPM and in the Epstein–Zin–Weil models is consistent with the estimates in Wheatley (1988) and Breeden et al. (1989). Wheatley uses a monthly sample from 1959 to 1981 and examines 40

[19] This evidence contrasts with the results in Mankiw and Shapiro (1986). They ran the empirical equivalent of the Epstein–Zin–Weil model and found that market beta is a priced factor, with a large positive t-statistic, but that consumption beta is not. Mankiw and Shapiro restricted their sample of firms to those present in the CRSP tapes every year during the period 1959–1982. Fama (1991) postulates that such sample selection leads to survival bias.

[20] Breeden et al. (1989) argue that temporal aggregation biases the estimate of γ upward.

stock portfolios ranked annually by the previous five-year return, a government bond portfolio, and a corporate bond portfolio. He estimates a γ of 139 when the consumption data include only non-durables, and a γ of 324 when the consumption data, like ours, include non-durables plus services. Breeden et al. examine twelve industry portfolios, the CRSP value-weighted portfolio, and four bond portfolios over the period 1959–1982. Breeden et al. do not provide explicit estimates of γ, but one can derive an implied estimate from their Table 1 (p. 240) and Table 5 (p. 253). Ignoring the serial correlation of consumption growth, their implied γ estimate is approximately 145 with quarterly data, and 335 with monthly data.

Wheatley suggests that high estimates of γ may be due to the fact that in the consumption-CAPM, the coefficient of relative risk aversion is also the inverse of the coefficient of intertemporal substitution. Our results show that the argument may have little merit since our estimate of γ actually *rises* from 121 in the consumption-CAPM to 294 in the Epstein–Zin–Weil model, which does not impose the restriction $\gamma = 1/\sigma$. The same conclusion may be reached based on Epstein and Zin's estimation (Epstein and Zin, 1991). Fama (1991) postulates that the positive relation between expected returns and consumption betas in Wheatley and Breeden et al., and their high γ estimates, may come primarily from the spread between bonds (low betas and low average returns) and stocks (high betas and high average returns). Our evidence here shows that the positive relation between average return and consumption risk, as well as high estimates of γ, are equally present in a restricted sample of assets that includes only stocks. [21]

So far, the combined evidence in favor of Campbell's model is weak. Although estimating the model derives what some authors have construed to be reasonable estimates of the representative agent's preference parameters, [22] the model fails in practice to uphold its implied parameter restrictions, which are the main theoretical contributions of the model. Specifically, the risk associated with innovations in the state variables is priced in a way that differs from the restrictions that Campbell's two-factor (heteroskedastic) model imposes. When we do not include measured consumption as a state variable, the model's overidentifying (orthogonality) conditions are rejected. When we do allow consumption as a state variable in the

[21] Contrary to our findings and the findings of Wheatley (1988) and Breeden et al. (1989), Epstein and Zin (1991) and Hansen and Singleton (1983) provide estimates of γ that are close to unity (logarithmic preferences). Both of these papers, however, estimate γ within a model that relates the level of consumption growth to the level of asset returns. Wheatley provides simulation evidence, which shows that in such a set-up, measurement error in consumption results in very imprecise estimates of γ and, moreover, severely biases the γ estimate downward.

[22] Estimating a "low" level of the coefficient of risk aversion should not be used in isolation to judge the economic meaning of an economic model; some recent studies entertain traditionally high levels of risk aversion (Abel, 1994, Campbell and Cochrane, 1995). We point out our lower estimate of γ in the Campbell model to place it in the context of the earlier literature mentioned above.

VAR, we find that although the overidentifying restrictions are not rejected, the parameter restrictions of the two-factor model continue to be rejected. Nonetheless, empirical models based on using measured consumption growth as an explicitly priced factor perform even more poorly, also failing to satisfy the overidentifying restrictions. Also, a general model which nests both Campbell's approach and a consumption risk factor suggests that consumption risk adds no marginal explanatory power for expected returns over and above Campbell's specification.

4.3. *Accounting for cross-sectional variation in returns*

Any well-specified asset pricing model should account for both the cross-sectional and time-series variation in expected returns. Conversely, a model's failure to explain either the cross-sectional or time-series variation in expected returns, even though it explains the other, may lead to a general rejection of the model. Thus, in a system of equations which involves both time-series and cross-sectional elements, general tests of overidentifying restrictions and parameter restrictions lend little insight into where the model is failing. A natural question that arises is whether Campbell's model fails the statistical tests because it cannot adequately explain the cross-sectional variation in excess returns across the size portfolios, that is, it cannot explain the "size effect".

A possible approach is to estimate the system of equations allowing each portfolio equation to have a different intercept (an additive constant). Typically, the presence of a significant intercept term provides an alternative specification test of an asset pricing model (Gibbons et al., 1989). Alternatively, a non-zero constant term may be interpreted as the outcome of an approximation error in the linearization. In any case, the presence of significant (non-zero) constant terms suggests that the included three factors do not provide an adequate representation of the cross-sectional dispersion of average risk premia. Although there may be no simple way to rigorously separate the time-series and cross-sectional implications of the Campbell model (and we do not presume that a model with separate intercepts can be derived as a general case using Campbell's framework), intuitive empiricism suggests that allowing for different intercepts effectively loosens the requirement that the Campbell model explain both the time-series variation *and* the average cross-sectional variation in risk premia.

In Table 5, we re-estimate the models of Table 4 allowing for separate portfolio intercepts $(a_1, \ldots, a_{10})$ in each of the ten model equations describing portfolio returns. As before, we estimate a set of ten portfolio equations plus a four-variable VAR system (RRET, DYLD, DTBL, GCON) and all models are estimated using five instruments, the first-lagged values of the state variables in the VAR plus a constant. This set of instruments results in 70 $(=[10+4]\times 5)$ orthogonality conditions.

In Panel A, we estimate the unrestricted four-factor model. The results contrast

Table 5

Intertemporal asset pricing models: GMM estimates from a four-variable VAR and ten size-portfolio equations with separate additive constants [a]

Panel A: Unrestricted four-factor model

$v_{i,t+1} = R_{i,t+1} - R_{f,t+1} - a_i - b_{\mathrm{RRET}}(w_{\mathrm{RRET},t+1} r_{i,t+1}) - b_{\mathrm{DYLD}}(w_{\mathrm{DYLD},t+1} r_{i,t+1}) - b_{\mathrm{DTBL}}(w_{\mathrm{DTBL},t+1} r_{i,t+1}) - b_{\mathrm{GCON}}(w_{\mathrm{GCON},t+1} r_{i,t+1})$; $i = 1,\ldots,10$

Parameter estimates and t-statistics								Model tests and statistics	
b_{RRET}	b_{DYLD}	b_{DTBL}	b_{GCON}	λ_{RRET}	λ_{DYLD}	λ_{DTBL}	λ_{GCON}	$J_\Phi(36)$	H0: $\lambda_{\mathrm{RRET}}/b_{\mathrm{RRET}} = \lambda_{\mathrm{DYLD}}/b_{\mathrm{DYLD}} = \lambda_{\mathrm{GCON}}/b_{\mathrm{GCON}}$; $\Phi_W(2) = 4.56$ [0.10]
−6.56	216.5 *	−103.2 *	113.4 *	0.01	8.55 *	−0.85	−1.16	41.6	H0: $a_1 = a_2 = \ldots = a_{10}$; $\Phi_W(9) = 37.3$ [0.00]
(1.46)	(3.62)	(−5.47)	(3.02)	(0.00)	(2.00)	(−1.31)	(−0.98)	[0.24]	
$a_1 = 0.114$ (4.10), $a_2 = 0.107$ (4.11), $a_3 = 0.102$ (4.33), $a_4 = 0.092$ (4.18), $a_5 = 0.089$ (4.12)								-	H0: $a_1 = a_2 = \ldots = a_{10} = 0$;
$a_6 = 0.088$ (4.23), $a_7 = 0.086$ (4.43), $a_8 = 0.076$ (4.24), $a_9 = 0.068$ (4.06), $a_{10} = 0.053$ (3.56)									$\Phi_W(10) = 54.4$ [0.00]

Panel B: Campbell two-factor model under heteroskedasticity ($\psi \neq 0$)

$v_{i,t+1} = R_{i,t+1} - R_{f,t+1} - a_i - b_M(w_{1,t+1} r_{i,t+1}) - b_{SC}\Sigma_{k=1}^{K}(\lambda_k w_{k,t+1} r_{i,t+1})$; $i = 1,\ldots,10$

Parameter estimates and t-statistics						Model tests and statistics	
b_M	b_{SC}	λ_{RRET}	λ_{DYLD}	λ_{DTBL}	λ_{GCON}	$J_\Phi(38)$	H0: $b_{SC} = b_M - 1$; $\Phi_W(1) = 23.0$ [0.00]
1.77	21.67 *	0.02	10.94 *	−0.88	0.25	46.6	H0: $a_1 = a_2 = \ldots = a_{10}$; $\Phi_W(9) = 34.3$ [0.00]
(0.58)	(4.35)	(0.00)	(3.70)	(−1.20)	(0.26)	[0.16]	H0: $a_1 = a_2 = \ldots = a_{10} = 0$; $\Phi_W(10) = 57.4$ [0.00]

Panel C: Campbell one-factor model

$v_{i,t+1} = R_{i,t+1} - R_{f,t+1} - a_i - b_M(w_{1,t+1} r_{i,t+1}) - (b_M - 1)\Sigma_{k=1}^{K}(\lambda_k w_{k,t+1} r_{i,t+1})$; $i = 1,\ldots,10$

Parameter estimates and t-statistics					Model tests and statistics	
b_M	λ_{RRET}	λ_{DYLD}	λ_{DTBL}	λ_{GCON}	$J_\Phi(39)$	H0: $a_1 = a_2 = \ldots = a_{10}$; $\Phi_W(9) = 83.3$ [0.00]
7.87 *	0.00	3.20 *	0.33	0.19	52.1	H0: $a_1 = a_2 = \ldots = a_{10} = 0$;
(4.12)	(0.00)	(2.42)	(0.36)	(0.30)	[0.08]	$\Phi_W(10) = 89.0$ [0.00]

Panel D: Epstein–Zin–Weil model

$v_{i,t+1} = R_{i,t+1} - R_{f,t+1} - a_i - b_M(w_{\mathrm{RRET},t+1} r_{i,t+1}) - b_{\mathrm{GCON}}(w_{\mathrm{GCON},t+1} r_{i,t+1})$; $i = 1,\ldots,10$

Parameter estimates and t-statistics			Model tests and statistics		
b_M	b_{GCON}		$J_\Phi(38)$	H0: $b_M + b_{\mathrm{GCON}} = 1$ (logarithmic utility); $\Phi_W(1) = 23.0$ [0.00]	H0: $a_1 = a_2 = \ldots = a_{10}$; $\Phi_W(9) = 95.8$ [0.00]
5.74 *	90.0 *	52.5	52.5		H0: $a_1 = a_2 = \ldots = a_{10} = 0$;
(2.84)	(4.49)	[0.06]	[0.06]		$\Phi_W(10) = 101.3$ [0.00]

Panel E: Campbell two-factor model augmented with consumption factor

$r_{i,t+1} = R_{i,t+1} - R_{f,t+1} - a_i - b_M(w_{1,t+1} r_{i,t+1}) - b_{SC}\Sigma_{k=1}^{K}(\lambda_k w_{k,t+1} r_{i,t+1}) - b_{GCON}(w_{GCON,t+1} r_{i,t+1})$; $i = 1,\ldots,10$

Parameter estimates and t-statistics							Model tests and statistics	
b_M	b_{SC}	b_{GCON}	λ_{RRET}	λ_{DYLD}	λ_{DTBL}	λ_{GCON}	$J_\Phi(37)$	H0: $a_1 = a_2 = \ldots = a_{10}$; $\Phi_W(9) = 37.0$ [0.00]
−6.23	56.3 *	203.1 *	0.01	4.22	−1.88 *	−1.60	41.2	H0: $a_1 = a_2 = \ldots = a_{10} = 0$;
(−1.11)	(3.50)	(2.13)	(0.00)	(1.18)	(−2.62)	(−1.59)	[0.29]	$\Phi_W(10) = 54.5$ [0.00]

Panel F: Static CAPM and consumption CAPM

$r_{i,t+1} = R_{i,t+1} - R_{f,t+1} - a_i - b_k(w_{k,t+1}, r_{i,t+1})$; $k =$ RRET, GCON; $i = 1,\ldots,10$

Parameter estimates and t-statistics		Model tests and statistics
b_M	b_{GCON}	$J_\Phi(39)$
6.89 *	-	52.23 *
(3.91)		[0.08]
-	120.6 *	53.42 *
	(6.77)	[0.06]

[a] Results are based on 130 observations (1959:3–1991:4). The excess return on portfolio i, $R_{i,t+1} - R_{f,t+1}$, is calculated as the discrete return from the end of quarter t to the end of quarter $t+1$ minus the annualized three-month bond-equivalent T-bill yield at the end of quarter t divided by four. In Panels A, B, C, D, E, and F, the ten portfolio equations (with separate additive constants) are estimated jointly with the four-variable VAR (Table 2, Panel C) using the generalized method of moments and allowing for conditional heteroskedasticity (case (ii) of Hansen, 1982, p. 1043). The GMM instruments are: constant, $RRET_t$, $DYLD_t$, $DBLT_t$, and $GCON_t$, resulting in $5 \times (10 + 4) = 70$ orthogonality conditions. * indicates significance at the 5% level and t-statistics are reported in parentheses under the parameter estimates. J_Φ (d.f.) is Hansen's (1982) χ^2-statistic with degrees of freedom equal to the number of orthogonality conditions minus the number of estimated parameters. Φ_W is a Wald χ^2-statistic associated with the specified parameter restrictions and is based on the unrestricted model. Probability values for J_Φ (d.f.) and Φ_W (d.f.) are reported in square brackets under the model test statistics.

with the model of Panel A in Table 4. First, the J_{Φ} statistic reveals that the overidentifying restrictions are not rejected (p-value = 0.24). Secondly, the price attached to DTBL risk is negative, which is consistent with the negative (albeit insignificant) estimate of the risk weight λ_{DTBL} implied from the VAR. As a result, we marginally fail to reject the test of the parameter restrictions implied by Campbell's two-factor model under heteroskedasticity (p-value 0.10). A test of the null hypotheses that the estimated additive constants are equal to one another (or all equal to zero) rejects strongly, suggesting that the four-factor model cannot account for the size effect.

In Panel B, we impose the restrictions of the Campbell two-factor model under heteroskedasticity directly, while allowing for separate additive constants. Again, the model marginally fails to reject the overidentifying restrictions (p-value 0.16). Estimates of the portfolio intercepts and corresponding t-statistics are unreported but are similar to those of Panel A. Tests of the restrictions on the portfolio intercepts suggest that the null of equality and the null that they are all zero are rejected. The table suggests that much of the cross-sectional variation in asset returns can be explained by the market factor V_{1m}, since allowing the portfolios to have separate intercepts greatly reduces the price of market risk b_m. Note, however, that b_{SC}, the price attached to the second risk factor $V_{i,h}$ remains positive and significant, suggesting that it continues to play a role in the intertemporal variation of expected returns.

Panel C estimates the Campbell model imposing the restriction that $b_{SC} = 1 - b_M$. Here, we find a coefficient of relative risk aversion equal to 7.86, which is both reasonable and statistically significant. However, statistical tests marginally reject the overidentifying restrictions of the Campbell model.

The Epstein–Zin–Weil two-factor model with actual consumption data is estimated in Panel D of Table 5. Hansen's J_{Φ} statistic marginally rejects the overidentifying restrictions of the model, and J_{Φ} is larger in Panel D than in Panel B, suggesting that the two-factor Campbell model adheres more closely to the orthogonality conditions. However, both factors in the Epstein–Zin–Weil model are positively priced and significant, with an implied estimate of γ equal to $b_M + b_{\mathrm{GCON}} = 95.74$ (standard error = 19.74). The implied estimate of the elasticity of intertemporal substitution remains close to zero ($\sigma = (1 - b_M)/b_{\mathrm{GCON}} = -0.053$).

Panel E presents the estimates of a three-factor model that augments the two-factor Campbell model with the measured consumption factor. Observe that in contrast to Panel E of Table 4, the consumption factor has a significant positive price; the Campbell two-factor model does not remove the explanatory power of the measured consumption factor in the augmented model. Finally, Panel F estimates the specification of the static CAPM and the consumption CAPM. As before, Hansen's J_{Φ} statistic continues to marginally reject the overidentifying restrictions of each model.

5. Summary and conclusions

The paper brings together under the unifying framework of Campbell's model two separate and extensive literatures in finance (Campbell, 1993a): the literature on the predictability of aggregate stock returns, and the literature on multiple-factor models of asset pricing with explicit macroeconomic factors. This unification occurs because Campbell's model substitutes innovations in consumption out of the model, replacing them with innovations in economic variables that help predict the return on invested wealth. Our analysis isolated two state variables that are robust predictors of quarterly real stock returns, the dividend yield and the nominal T-bill yield. To complete the specification we added the real aggregate stock return and consumption growth, leading to a total of four state variables. In the manner suggested by Campbell, we then used these variables as priced risk factors to explain the time-series and cross-sectional variation in the risk premia of ten size portfolios of U.S. stocks. The covariances of the portfolio real returns with innovations in these state variables serve as the factors in our multiple-factor framework.

Our empirical analysis examined the performance of both the unrestricted multi-factor model and several restrictive versions of the conditional form of Campbell's model, most notably: (1) the two-factor Campbell model under heteroskedasticity and (2) the one-factor Campbell model. We also compared the performance of these models to empirical models which use consumption as an explicitly priced risk factor, such as the two-factor model of Epstein and Zin, 1989, 1991 and Weil, 1989, 1990, and one-factor models such as the static CAPM and the consumption CAPM. In the course of the analysis, we made numerous assumptions, such as the assumption that the difference between the nominal T-bill yield and the actual inflation rate is a risk-free real interest rate, and the assumption that wealth is proxied by the market portfolio. These assumptions are not directly related to Campbell's basic proposal, which remains a theoretical construct. The assumptions were made to facilitate estimation, however, and to allow comparison with other studies.

The initial results do not provide support for Campbell's proposal to substitute consumption out of the representative-agent model. We find that a high dividend yield is associated with higher future investment opportunities, and portfolios that covary positively with the dividend yield are compensated with a higher expected return. However, a high T-bill yield is typically associated with *lower* future investment opportunities, yet portfolios that covary *positively* with the T-bill yield also command a higher expected return. This is inconsistent with the restrictions of Campbell's model, and formal tests show that the parameter restrictions of the two-factor Campbell model are rejected. However, the specification tests of Hansen (1982) reveal that compared to empirical models that explicitly price the consumption factor, the Campbell model more closely adheres to the implied orthogonality conditions. When consumption is introduced as a state variable, the model's overidentifying orthogonality conditions are upheld, but the estimation again leads to a rejection of the theoretical model's implied parameter restrictions. Compared to models that explicitly price consumption risk, Campbell's two-factor model leads to lower implied estimates of the coefficient of relative risk aversion. Apparently, the variability of "true" consumption – implied by the joint variability of the state variables that substitute consumption out of the representative agent model and the sensitivity parameters λ_k – is much higher, leading to lower (and possibly more realistic) values of the parameter of relative risk aversion (see also Campbell, 1993b).

Augmenting the model by allowing a separate additive constant for each portfolio in the empirical model helps focus the subsequent analysis on the time-series properties of the models. In this case, the estimation reveals that neither the Campbell model, nor the models based on the explicitly priced consumption factor explain the "size" effect. Nonetheless, statistical tests using the augmented empirical model reveal that the parameter restrictions of the Campbell model are now (marginally) upheld in the time-series. Estimating Campbell's augmented two-factor model reveals that the orthogonality restrictions are not violated, while the augmented model continues to provide lower estimates of the risk aversion parameter γ than models that do not substitute consumption out of the empirical risk-return relationship. Unlike the augmented Epstein–Zin–Weil model and the augmented consumption-based CAPM, the overidentifying restrictions of Campbell's augmented two-factor model extended to allow separate portfolio intercepts are not rejected.

6. For further reading

Fama and French (1988), Ferson (1989) and Poterba and Summers (1988).

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