ARE RATIONAL EXPECTATIONS FOR REAL?

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Introduction and Overview

The concept of rational expectations is intuitively appealing because it appears to be an application of the fundamental principle of individual optimizing behavior that underlies all economic theory. When the payoff from a decision depends on unknown future events, the decision maker, implicitly or explicitly, assigns subjective probabilities to the events that may affect the payoff. The outcomes of decisions are observable, but the subjective beliefs of individuals are unobservable. As a result, without a restriction on individual subjective beliefs any decision can reflect optimal behavior, and the theory of optimal decision making under uncertainty is empty. Even psychotic behavior can be optimal given a sufficiently distorted perception of the world.

John Muth, in a now famous article, suggested a restriction on individual subjective beliefs that appears to follow naturally from the assumption of individual optimizing behavior. Muth proposed that individuals' subjective probabilities should equal the observed frequencies of events or the «objective» probabilities[1]. Muth coined the term «rational expectations» to describe the restriction.

Since the publication of Muth's paper, it has been shown that the assumption of rational expectations has widespread theoretical implications. Most of the literature on rational expectations demonstrates that replacing *ad hoc* forecasts, e.g. adaptive expectations, with rational expectations forecasts, e.g. the conditional mean of the «objective» probability distribution, dramatically alters the results and the economic interpretation of a model. In macroeconomics, models with no long-run money illusion have no short-run money illusion when expectations are rational. Although many economists view the short-run theoretical conclusions of rational expectations models

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1. Muth, p. 316. Muth only equates the average of individual's subjective beliefs with the objective probability. For the purposes of this paper, the distinction between the average and specific individuals makes no substantive difference.

with skepticism, the conclusions follow logically and inescapably from the properties of the models, the definition of rational expectations, and the assumption that individuals optimize.

This paper examines the link between the intuitive notion of expectations based on optimization and the specific definition of rational expectations. We ask: when does optimization imply that individuals equate subjective and «objective» probabilities, i.e. when are rational expectations a corollary of the principle of optimization? A necessary condition is that individuals must be able to infer the «objective» probabilities[2] from observable data – rational expectations must be operational.

Equating subjective probabilities with objective probabilities raises a conceptual and a practical issue. The conceptual issue is whether all economic events have well-defined objective probabilities. For objective probabilities to exist at any point of time, there must be, at least conceptually, a universe of realizations. Classical probability theory is based on a frequency interpretation of probability, and classical inference is based on replication under the same environment. As the number of replications gets large, the estimate of the frequency, or the estimator of some moment of the probability distribution, converges to the limiting relative frequency, or the true moment.

The practical issue is whether observable data contain sufficient repetitions to accurately infer the frequencies, or the objective probabilities, of economic events. While replication is possible in physical experiments, replication may not be characteristic of economic processes. If economic agents have free will and the ability to change their environment, replication of economic experiments under the same environment is ruled out. Powerful policy makers and cartels are examples of groups of agents who may alter the circumstances under which the rest of the population makes economic decisions.

The notion that economic events may not be replicable is not new in economics. Frank Knight defined risk as a situation where replication occurs and randomness could be objectively quantified (a known probability distribution) and uncertainty as a situation in which randomness could not be quantified. J.M. Keynes (1948, p. 185) made a similar distinction. He said, «The game of roulette is not subject, in this sense, to uncertainty... The sense

2. Bayesians argue that the frequency interpretation of probability is inconsistent and that the objective probability of any event is not well defined, e.g. see Leamer. Bayesians substitute the notion of consensus beliefs to describe the probability of an outcome in similar repeated experiments like the draw of a card. Following Muth, we will use the term objective probability. The distinction between consensus beliefs and limiting frequencies or objective probabilities is essentially semantic in the context of this paper.

in which I am using the term is that in which the prospect of a European war is uncertain...».

In our framework risk describes random but recurrent events. The frequency of the event can be estimated and the randomness objectively quantified. Uncertainty describes events generated in a novel environment – the outcome is unknown and therefore random from the individual's perspective; but there is no past history to objectively quantify the randomness. For an individual the distinction between risk and uncertainty is not relevant; the individual makes a «best» subjective evaluation and a decision. For rational expectations and aggregate economic modelling the distinction is relevant.

Robert Lucas (1977, p. 15) clearly states the distinction between risk and uncertainty and concludes that:

> In situations of risk, the hypothesis of rational behavior on the part of agents will have usable content so that behavior may be explainable in terms of economic theory. In such situations, expectations are rational in Muth's sense. In cases of uncertainty (neoclassical?)[3] economic reasoning will be of no value.
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> These considerations explain why business cycle theorists emphasize the *recurrent* character of the cycle, and why we must *hope*[3] they were right in doing so. Insofar as business cycles can be viewed as repeated instances of essentially similar events, it will be reasonable to treat agents as reacting to cyclical changes as "risk", or to assume that their expectations are *rational*.

The key practical issue *is* whether or not actual observable data reveal a recurrent pattern; this is a testable hypothesis. If the data reveal recurrent patterns, then it is reasonable to model agents' behavior as if they were making decisions under risk; if not, then it is reasonable to treat agents' behavior as if they were making decisions under uncertainty and to change our economic models. Recently P.A.V.B. Swamy, James Barth, and Peter Tinsley (1982) presented a detailed description of the Bayesian and classical concepts of probability as they relate to rational expectations. They concluded that «the coincidence of subjective and objective probabilities of outcomes, is too restrictive to be very useful in practice».

In a rational expectations world, where economic agents operate in an environment of risk rather than uncertainty, there are no truly discretionary policies that can be chosen by a policy maker. If there are some free parameters whose values can be chosen, economic agents in rational expectations models will assign a subjective probability distribution over these parameters which equals the assumed objective probability distribution, thus making these parameters no longer free. Free will is, therefore, ruled out or relegated

3. Parenthetical comment and emphasis added.

to a white noise error term. This description of individual decisions implies a philosophy of human behavior as a sort of stochastic Calvinism. It suggests that the systematic component of individual decisions is predestined by the parameters of a time invariant probability distribution. Thomas Sargent and Neil Wallace (1976) admit that this is a «conundrum» facing the economist who builds rational expectations models.

The lack of truly discretionary policies in rational expectations models is essentially the complement of Lucas' (1976) criticism of traditional macroeconometric policy simulation results. Lucas argued that traditional macroeconometric policy simulation results were incorrect because the models on which they were based implicitly assumed that individuals ignore available information when forming expectations. Lucas pointed out that if a policy were changed, the private sector would observe the change and modify its expectations. Traditional proxies for expectations, e.g. adaptive expectations, omit observable exogenous conditioning information. The rational expectations formulation includes the conditioning variables. However, it does not address the issue that future values of the conditioning variables are not currently observable information and that, therefore, individuals must also form expectations about the conditioning variables. Here the issue of risk versus uncertainty is crucial. If the policies and other exogenous variables reflect free will decisions, then they can change the underlying environment, and rational expectations are not operational. If the policies are drawn from a fixed objective probability distribution, then rational expectations are operational, but there are no true discretionary policy decisions[4].

In Section 1 of this paper we focus on the theoretical issues involved in forming rational expectations. A necessary condition for optimization to imply rational expectations is that the vector of endogenous and exogenous variables in the economy is a jointly stationary stochastic process, or that the form of the nonstationarity is known. A process is called stationary if its probability structure does not change with time. Stationarity provides the link to equate subjective and objective probability through a commonly observed data set that draws individuals with diverse subjective prior beliefs toward the consensus or objective rational expectations belief. In a nonstationary environment the data are realizations from different regimes and cannot bridge the gap between subjective and objective probability.

Section 2 examines the hypothesis that rational expectations are operational at a practical level. The practical issue is the most important issue.

4. Lucas (1976), pp. 33-35, discusses policy regime changes as draws from an objective probability distribution and notes the problem in evaluating «ad hoc» rule changes.

It makes little difference if the true economic process is nonstationary, if models with fixed parameters yield reasonably accurate approximations. By the same token, the belief that the true process is stationary – but impossibly difficult to accurately parameterize with the observable data set – provides little comfort to a practical forecaster and little assurance that the estimated probabilities will be stable and trusted.

We present the results of empirical tests for parameter stability using approximately one hundred years of U.S. data. These tests require very few assumptions about the true structure under the null hypothesis. The data reject the hypothesis that the same structure can explain both halves of the sample. While this is not a test of stationarity – which is nontestable – it has the same practical implication for individual verification of subjective expectations. The data do not form a basis for consensus rational expectations beliefs.

Section 3 contains a summary and our conclusions.

Section 1: Theoretical Considerations

The central theoretical issue for rational expectations involves the ability of optimizing individual agents to equate their subjective beliefs with the objective probability of an event. This section illustrates the crucial role of the stationarity assumption in making rational expectations operational. Stationarity in economic process implies that economic events repeat themselves, although the pattern may be complicated[5]. Rational behavior implies that an individual does not repeatedly make the same mistake. Taken together stationarity and rational behavior guarantee that learning from the past is feasible and rational expectations are operational[6].

Although physicists and engineers made spectacular theoretical and

5. A process is stationary if its probability structure does not change with time. Space (cross sectional) averages of functions of a stationary process are constant. A stationary process is ergodic if time averages converge to the corresponding space averages. A process may be stationary but nonergodic if the probability structure depends on an initial draw. Thus, there could be many stationary processes each dependent on the initial draw. The time averages would depend on the initial condition. An ergodic process has sufficient independence so that initial conditions are unimportant. Since we are interested in a change in the process we concentrate on stationarity. See Malinvaud (1966), pp. 387-388 for a discussion of ergodicity.

6. This paper does not address the technically complex issue of aggregate learning. We are interested in the existence of a rational expectations equilibrium, and do not consider the problems in reaching an equilibrium.

practical advances using equations of motion to model inert physical processes as stationary stochastic processes, it is not obvious that the same methodology is always productive in economic modelling. Physical processes are subject to constant "laws of nature," so the fixed parametric structure of a stationary-stochastic process is an accurate description of reality for most problems. Economic outcomes, however, depend on human behavior as well as physical events. Although individual decisions can be modelled as realizations from a stationary stochastic process, the representation may not be a reasonable approximation. Stationarity implies that an individual facing the same set of circumstances at different times will make the same decision except for a purely random component. This is stochastic Calvinism. Free will is relegated to a white-noise error.

It can and should be argued that the stationarity assumption is not as stringent when applied to an average of individual decisions. After all, markets average or aggregate many individual decisions so that an "aberrant" individual decision has virtually no effect on the outcome in a competitive market. This is a sort of central limit theorem applied to individual decisions: when individual decisions are averaged through the market and as market participation gets large, the cross-sectional average of the individual time-varying distributions converges to a limiting distribution with fixed parameters[7].

Under the fairly general conditions for a central limit theorem market averaging of independent individual decisions yields a conditional probability distribution, $P(\underline{y}/\underline{x})$, for the endogenous, market-determined, variables, $\underline{y}$, given the exogenous variables, $\underline{x}$, which has fixed parameters. In principle, the parameters of this distribution can be consistently estimated with observable data. Knowledge of the parameters of the conditional distribution, however, is not sufficient for prediction unless the exogenous variables are known. When future exogenous variables are unobservable, they also must be predicted. Therefore, predictions of the future are predictions from the joint distribution:

$$P(\underline{y},\underline{x}) = P(\underline{y}/\underline{x})P(\underline{x}).$$

The exogenous variables include policy variables (or policy rules) and cartel policies (OPEC) which unlike physical processes can be changed at

7. Frydman (1982) provided examples where individual agents cannot learn the constant parameters of the equilibrium distribution. Our argument is about existence and not about learning.

the discretion (free will) of a decision maker. Unlike endogenous variables they are not averaged by a market, so that there is no "central limit" argument to justify a time invariant probability structure. If some of the players (decision makers or nature) can change the rules of the game (the policy rule, the terms of trade, or the weather pattern), the process is nonstationary. In a nonstationary environment an individual cannot verify his or her subjective expectations of future events with data. When future events are drawn from a different probability distribution, the past is not a reliable guide to the future. Even in the case of an announced exogenous change, say an announced policy regime shift, the private sector must re-evaluate its subjective expectations. Economic agents must decide whether the change is temporary or permanent or whether the announcement signals any change at all.

This section examines the feasibility of equating subjective beliefs with objective probabilities at a theoretical level. Part 1.1 presents a popular macroeconomic rational expectations example. Its purpose is to emphasize that future values of the exogenous variables affect current expectations. Part 1.2 shows that if the joint process for the endogenous and exogenous variables is stationary, then it is feasible for an individual to equate subjective probabilities with objective probabilities. Parts 1.3 and 1.4 consider changes in the exogenous process. Part 1.3 shows that if the change is unique there is no data to form the basis for a consensus rational expectations belief. Part 1.4 considers the alternative that regime changes are random events drawn from a stable distribution so that rational expectations are feasible.

1.1 A Macroeconomic Example

The macroeconomic illustration uses the rational expectations version of the model in the Sargent and Wallace (1975) article. The structure is described by the following equations:

(1.1.1) $$y_t = b_{11}k_{t-1} + b_{12}(p_t - p_t^{RE}|_{t-1}) + u_{1_t}$$ A «Lucas» supply curve

(1.1.2) $$r_t = b_{21}y_{t-1} + b_{22}k_{t-1} + b_{23}x_t + u_{2_t}$$ An «IS» equation

(1.1.3) $$k_t = b_{31}r_t + b_{32}k_{t-1} + b_{33}x_t + u_{3_t}$$ A capital accumulation equation

(1.1.4) $$p_t = m_t + b_{42}y_t + b_{43}i_t + u_4$$ An «LM» equation

(1.1.5) $$i_t = r_t + (p_{t+1|t-1}^{RE} - p_{t|t-1}^{RE})$$ The Fisherian nominal interest rate where y, k, p, m are the natural logarithms of real income, the capital stock, the price level, and the nominal money stock, respectively. The variable r is the Fisherian real interest rate, i is the nominal interest rate, x are exogenous variables, and the u_j are Gaussian white-noise errors. Expectations are rational so p^{RE} is the conditional mean of the objective probability distribution for p.

When expectations are rational the model has a simple recursive structure which can be solved for the rational expectation of the price level:[8]

$$p^{RE}_{t|t-1} = \sum_{j=0}^{\infty} c^j \, E\left(m_{t+j} + ax_{t+j} \middle| t-1\right) \tag{1.1.6}$$

The conditional expectation of the price level depends on *future* values of both the money stock and x – the exogenous variables. To evaluate equation (1.1.6) the exogenous variables must be known or the stochastic process generating the variables must be known. In the model only unanticipated changes in the nominal variables, i.e. curent error realizations affect the real variables.

1.2 Equating Subjective Probability with Objective Probability

Rational expectations assumes that individuals know the joint objective probability distribution for the endogenous and exogenous variables. Given the objective probability distribution and the assumption that individuals optimize, individuals make minimum mean square error forecasts which are uncorrelated with any conditioning information.

The fundamental issue for rational expectations is not related to the properties of optimal forecasts. The real issue involves the ability of individuals to equate their subjective beliefs with the objective probability of an event. The assumption that individuals optimize guarantees that they make optimal forecasts given their subjective beliefs; but it does not guarantee that they will be able to equate their subjective beliefs with the objective probability.

If economic time-series are stationary stochastic processes, then it is feasible for an individual to equate his subjective beliefs with the objective probabilities. Furthermore, optimization implies that an individual will use observable data to verify his subjective beliefs and, therefore, rational expectations follow as a corollary of optimization.

Let the objective probability distribution of the vector of endogenous

8. See Sargent and Wallace, p. 248, equation 20.

variables $\underline{y}$ and exogenous variables $\underline{x}$ be denoted by Θ. For simplicity assume that Θ can be uniquely characterized by the vector of parameters $\underline{\theta}$, so that individuals are able to equate subjective and objective probabilities when they learn the true parameters $\underline{\theta}$. When the i^{th} individual believes the true parameters are $\underline{\theta}^i$, this gives rise to a subjective distribution Θ^i. Based on Θ^i the individual's optimal minimum mean squared error forecasts are the conditional means $(\underline{y}^i, \underline{x}^i)'_{t+s}|_t$. As long as $\Theta^i \neq \Theta$ the individual's forecast errors will have a nonzero mean and a larger mean squared error than forecasts from the correct distribution. As a result an optimizing individual will adjust his prior belief until it agrees with the true distribution. This is the intuitive argument that rational expectations are a corollary of optimization.

The adjustment of prior subjective beliefs can be illustrated as a Bayesian estimation procedure. Suppose the investor begins with an initial subjective prior $f(\underline{\theta}^i)$. The investor's initial subjective prior, $f(\underline{\theta}^i)$, is a Bayesian prior which can be made without reference to the data. If he believes that future realizations will be drawn from the same probability distribution that produced the currently observable data, Z, he can compare his subjective prior belief with estimates from the data, e.g. the classical estimates that maximize the likelihood function, $L(\underline{\theta}| Z_t)$. A rational investor will modify his prior if it is inconsistent with the data. Formally, the agent mixes the data with the prior to form a posterior:

$$F(\underline{\theta}^i) \sim L(\underline{\theta} \,|\, Z_t)\, f(\underline{\theta}^i). \tag{1.2.1}$$

The posterior determines the parameters $\underline{\theta}^i$ in the agent's subjective probability distribution.

Assuming the distribution is approximately correct, as the sample gets large, the individual's posterior, $F(\underline{\theta}^i)$, converges to the true parameters $\underline{\theta}$[9]. Notice that the investor's final subjective probability distribution is data based, i.e. the moments are estimated. Without consulting the data there is no way for the individual to validate his initial subjective prior distribution. Assuming rationality is not equivalent to endowing individuals with clairvoyant power. An agent's initial guess, $f(\underline{\theta}^i)$, will almost certainly be incorrect; but a rational agent will modify the initial guess using observable data so that his final subjective expectations are consistent with the actual process gene-

9. Linear methods can be used to obtain consistent estimates of the first two moments of any wide-sense stationary-stochastic process. e.g. see Fuller, Chapter 6, so the distributional assumptions underlying the Bayesian update are of second order.

rating the data[10]. Inference based on the commonly observed data set provides the basis for the «equilibrium» interpretation of rational expectations models. In a stationary environment the likelihood function portion of the posterior, which is common to all agents, draws individuals with diverse subjective prior beliefs to the consensus rational expectations belief.

1.3 Non-stationarity

The probability structure of a non-stationary process by definition is time varying. As a result the unconditional probability structure cannot be estimated. Non-stationarity takes two basic forms: processes that change as a regular function of time, e.g. a trend that causes the mean of a process to grow with time; and, processes that change at a point in time, e.g. the OPEC price increase in 1974 that caused the average oil price to shift.

If one knows the source of the time variation, i.e. *a priori* conditioning information, then it is possible to estimate the properties of the conditional process. Econometrics emphasizes «structural» modelling and estimation techniques precisely because the structural parameters (moments of the conditional process) are invariant with respect to changes in the exogenous variables (the conditioning information). The estimated structural model can be used to make quantitative predictions of the effect of hypothetical exogenous changes or policy interventions on the endogenous economic variables. For forecasting or rational expectations, however, knowledge of the conditional distribution is insufficient. The conditioning variables, or the process generating the conditioning variables, must be known or estimated.

Economic time-series that are non-stationary but regular functions of time create no conceptual problems for rational expectations predictions. For example, many variables have systematic growth components (trends) or are believed to be random walks, so that the process is explosive and non-stationary. Detrending by differencing or quasi-differencing removes the non-stationary component, and the transformed series can be used to estimate the moments of the conditional process. Since the conditioning variable, e.g. a trend, is a regular function of time, observable data can be used to parameterize the conditioning process as a function of time.

Conceptual problems for rational expectations occur when the source of the non-stationarity is not a regular function of time, e.g. the 1974 (and

10. This is not a proof or an argument that if all agents are learning there will be aggregate convergence; the illustration is for an individual whose actions cannot affect the market outcome.

1979) increase in oil prices, or a change in the monetary regime or the tax structure. Individuals will form subjective beliefs about the probability of future changes, but unless the changes are regular, i.e., there is a stable frequency that can be measured, there is no way for an individual to verify his or her subjective beliefs. There is no commonly observed data set to form the base for a consensus belief.

To illustrate, consider an example where the exogenous process – which we will call the price of oil – is non-stationary. Agents know the price of oil increased and they believe it may increase again. That is, they calculate the probability of a regime shift. For simplicity, assume there are two possible regimes – either the average cost of oil will remain unchanged at χ, or it will be increased to χ^*. Thus an agent's subjective prior is :

$$x_t \sim \begin{cases} N(\chi, \sigma_v);\ 1-\pi^i \\ N(\chi^*, \sigma_v);\quad \pi^i \end{cases} \tag{1.3.1.}$$

The agent believes that the regime will be the same with probability $1-\pi$, and will change with probability π.

The problem lies in verifying the transition probability π^i with observable data. If the agent uses past annual oil prices (say from 1945 through 1975) the estimate of π will be very small (about 1/30) since there was essentially one regime shift in 1974. If he believes the cartel represents a new structure, then the data from the pre-cartel regime are of no value in estimating the probability of another price increase. Individual decision makers may well have different subjective evaluations of the probability of a regime shift. Agents make decisions under uncertainty in Knight's terminology.

The same argument applies to policy regime shift. Should the Reagan administration's policy announcements be accepted at face value? Are we moving to a permanent regime with lower government spending, lower taxes, lower money growth rates? Or will policies revert to the pre-1980 regime, or change to another new regime? The individual's evaluations of the transition probabilities and the potential regimes are subjective; and unless similar regime changes have occured frequently in the past there is no way to objectively quantify the risk. As a result, rational expectations are not well-defined.

1.4 Stationary Regime Changes

An alternative view is that there are no truly new or unique events in the world. Reaganomics, OPEC, Falklands (Malvinas), and birth control pills are simply realizations of specific policy regimes, cartels, wars, and technical progress. These events have all happened in the past and will occur again in the future. That is, the specific events are draws from a stable distribution.

To illustrate consider the example from section (1.3). There is always a probability that the average price of oil will increase, say π_1; but if the industry forms a cartel, the probability of a price increase is greater, say π_2. Thus. the probability of a price increase is conditional on the regime. However, if the formation and collapse of cartels are random events drawn from a stable distribution. then there is a mega distribution that nests the regimes:

$$\begin{array}{llr} & \left.\begin{array}{l} x_t \sim N(\chi, \sigma_v);\ 1-\pi_1 \\ x_t \sim N(\chi^*, \sigma_v):\ \pi_1 \end{array}\right\} & 1-\varphi \quad \text{no cartel} \\ (1.4.1) & & \\ & \left.\begin{array}{l} x_t \sim N(\chi, \sigma_v):\ 1-\pi_2 \\ x_t \sim N(\chi^*, \sigma_v);\ \pi_2 \end{array}\right\} & \varphi \quad \text{cartel} \end{array}$$

where φ is the probability that the industry will form a cartel.

The mega distribution for x is a mixture of normal and binomials. but the probability structure is fixed and the process is stationary. The marginal distribution for x has a constant mean:

$$E(x) = \chi^*\gamma + \chi(1-\gamma)$$

and constant variance:

$$E(x - E(x))^2 = (\chi^* - \chi)^2\gamma(1-\gamma) + \sigma_v .$$

where:

$$\gamma = \pi_1(1-\varphi) + \pi_2\varphi.$$

In a stationary environment an investor can use data to verify (estimate) his subjective beliefs about the marginal process for any variable or variables relevant for his decision since the process is stationary. In this example the knowledge that the OPEC cartel has formed is useful conditioning information for forecasting. but it does not change the probability that the cartel will remain in power.

Section 2: Empirical Evidence

Our empirical results shed light on the practical issue of whether or not rational expectations are operational. As shown in Section 1, it is the commonly observed data that draw optimizing individuals toward a consensus or rational expectations belief. If the data appear to be realizations from a stable distribution, then optimizing agents will use the data to verify their subjective beliefs. On the other hand, if the data appear to be realizations from different regimes, then individuals will ignore the data and rely on their own individual subjective beliefs when forming expectations; the data cannot be used to equate subjective and objective probabilities. In a stationary world, regime changes may occur, but the regime changes must be draws from a stable probability distribution. Every conditional random variable, is then nested in a stable mega distribution. Our intent is to test whether the historical aggregate time series, when detrended appear to be characterized by such a mega distribution with constant parameters. Our null hypothesis is that the detrended aggregate variables are covariance stationary processes[11].

Wold has shown that the indeterministic part of any covariance stationary process y_t has a linear representation[12],

$$y_t - \gamma = \sum_{j=0}^{\infty} \omega_j \, \varepsilon_{t-j}, \tag{2.1}$$

where γ is the mean, ω_j are constant parameters, and ε_{t-j} are white-noise errors. The linear representation (2.1) does not restrict the actual y_t process to be linear or Gaussian, nor does it imply that individuals form expectations (forecasts) using univariate ARMA (autoregressive – moving average) models. Indeed, sophisticated agents will use all their available information to condition their rational expectations predictions and reduce the variance of their forecast errors. This information set will often include other variables related to y_t besides its own past realizations. Equation (2.1) is not intended to capture the agent's forecasting rules. It is simply the linear representation of the covariance stationary process y_t, which is the aggregate outcome of many individuals' decisions who may have differing information sets. If the first two moments of the true joint mega process are independent of time, then the first two moments of any marginal process are also independent of time, and the parameters in (2.1) are constant. The constancy of the parameters in (2.1) is a necessary condition for the covariance stationarity of the joint mega process $(\underline{y}_t, \underline{x}_t)$, where $\underline{x}_t$ can be any vector of variables in the economy that are relevant in the individual decision making process.

11. We are assuming the first two moments of the process are finite.
12. See Anderson (1971) or Sargent (1979).

Focusing on the marginal process y_t does not restrict the generality of the results that follow.

The linear representation (2.1) of the covariance stationary process y_t is used to test the null hypothesis that rational expectations are operational at a practical level. Under the null hypothesis the parameters in (2.1) are constant. This suggests partitioning the sample and testing for stability of the parameters across the sub-samples. This is not a test for covariance stationarity *per se*, because a process may appear unstable in any finite sample when, in fact, the infinite sequence is covariance stationary. But it is a test of the null hypothesis that rational expectations are operational at a practical level. Parametric instability in (2.1) indicates that optimizing agents will not rely heavily on data when forming expectations[13]. From the point of view of the individual agents it makes no difference whether a series is non-stationary or simply appears to be so *ex post*, i.e. after they make a particular decision. In either case the data do not provide reliable information about the future.

When the moving average polynomial $\omega(L)$ in (2.1) is approximated by a rational polynomial, $\beta(L)/\alpha(L)$, we get the familiar ARMA model:

$$\alpha(L)\,(y_t - y) = \beta(L)\,\varepsilon_t \,. \tag{2.2}$$

We estimated univariate ARMA models for five annual U.S. time series: the real GNP (RGNP), the unemployment rate (U), the real wage rate (RWAGE), the real commercial paper rate (RCP), and an index of the capital stock (RCP). The data are described in detail in appendix 1.

The series were differenced or logarithmically differenced to remove systematic growth components. Models for the detrended series were identified and estimated using the techniques popularized by George E.P. Box and Gwilym M. Jenkins. The sample was then split into two portions – pre- and post-1929, and the models were estimated for each sub-sample.

Table 1 summarizes the results of likelihood ratio tests for parameter stability[14]. Column 4 contains the main results where all parameters are tested. Single asterisk indicates rejection of the null hypothesis at the 95 percent level and double asterisk at the 99 percent. The tests overwhelmingly reject the null hypothesis.

13. There is a qualification to the above statement: Agents should not have a longer data series than the econometrician. This is not a problem with our sample because it covers the period of reliable published data.

14. For more detailed results, see Appendix 2.

TABLE 1

Variable	ARIMA Identification	Sample Partition	$H_0\ (\theta_i = \theta_j)$	$H_0\ (\theta_i = \theta_j\ /\sigma^2)$	$H_0 = (\sigma_i^2 = \sigma_j^2\ /\ \{\alpha(L), \beta(L), \gamma\})$
ln RGNP	1,1,0	1889 - 1929	**	**	
		1930 - 1979			
RCAP	1,1,1	1889 - 1929	**	*	**
		1930 - 1979			
RCP	1,0,0	1890 - 1929	**		**
		1930 - 1979			
ln U	1,0,1	1890 - 1929	**	**	**
		1930 - 1979			
ln RWAGE	1,1,1	1862 - 1929	**	*	**
		1930 - 1979			

Note : (a) RGNP is real Gross National Product, RCAP is an index of real Capital Stock, RCP is inflation adjusted Commercial Paper Rate, U is Unemployment Rate, and RWAGE is real Wage Rate. (b) α is the set of autoregressive parameters, β is the set of moving average parameters, γ is the mean, σ is the standard deviation of the error term, and θ is the parameters α, β, γ, and σ. (c) Single asterisk denotes rejection of the null hypothesis at the .05 significance level, while double asterisk, at the .01 significance level. (d) The test of $H_0\ (\theta_i = \theta_j)$ is for all the parameters, the test of $H_0\ (\theta_i = \theta_j\ /\sigma^2)$ restricts σ^2 to be the same in the two sub-samples, and the test of $H_0\ (\sigma_i^2 = \sigma_j^2\ /\{\alpha(L), \beta(L), \gamma\})$ restricts all the ARMA parameters other than σ^2 to be the same.

The univariate ARIMA models, which were tested, are not unique approximations of the linear representations of the time series. To check the robustness of the results with respect to the use of univariate ARIMA models, a vector autoregressive model of the five detrended time series was also tested. The hypothesis of parameter stability was again rejected. Tables 7 and 8 of appendix 2 contain the detailed results.

The rejection of the hypothesis may be due to a type I error. But this seems rather implausible since the Great Depression was, in fact, followed by a change in economic theory and government and business policies that lasted forty years. The real world perceived the Great Depression as a structural break. Our results support this perception.

The choice of the break *point* at the beginning of the Great Depression was arbitrary, and it may be argued that the rejection was due to the fact that those years were a sequence of outliers; the pre-Depression and post-Depression years actually belong in the same regime, and by mere luck we chose the break point that would lead to a rejection of the null hypothesis. We also estimated the univariate ARMA models of the detrended series by partitioning the sample at the beginning of World War II. The hypothesis of parameter stability was again rejected. This indicates that the particular break point did not carry much weight in the rejection of the null hypothesis.

The misfortunes of the Great Depression years could not have been forecasted prior to its occurence. Yet, some may argue, there are other instances in our sample period when transient non-stationarities did occur that were easily forecastable, e.g. the World War II years. Accordingly, the sample should be purged of those easily recognized transient non-stationarities because they might lead to a false rejection of the null hypothesis. But purging non-stationarities is not easy in practice or conceptually. World War II seems obvious, but what about World War I, the Korean War, the Vietnam War, and the wage and price controls in the twenties, fifties, and the seventies? If many events are special cases, special cases will occur in the future, and unless the special cases are a stationary process they must be forecasted correctly to condition the rational expectations predictions.

Our sample covers a long time period. It was chosen for the purpose of checking for large changes in the economic environment which would have important effects on the economy. The hypothesis of strict stationarity however, may be too stringent. For practical purposes today, the most recent past is more relevant. It may be argued that the post World War II period was more stable and, therefore, rational expectations were an operational concept throughout this period. Although we have not performed any formal tests ourselves, Christopher Sims (1980), in another context, rejected the hypothesis of parameter stability for both U.S. and West Germany using a vector autoregressive model. He partitioned his post World War II quarterly sample at the period when both countries adopted flexible exchange rates. This leads us to the conclusion that our results are robust not only with respect to the models employed and the particular partitioning, but also with respect to the choice of the sample period.

Our results may be subject to a statistical criticism which arises from the fact that our sample, although it is long by traditional economic standards, may not be long enough. The sample distributions of the test statistics may not be well approximated by their asymptotic distributions. The actual distributions may have fat tails, so that the asymptotic normal or chi-square approximations understate the true variance leading to a false rejection of the null hypothesis. However, if the true distributions are stable but have fat tails, then individuals still have a difficult – if not impossible – task in verifying their subjective expectations with data. The parameters of fat-tailed distributions are notoriously difficult to accurately estimate with small data sets. If rational expectations are operational, then individuals in one way or another must estimate the parameters. When the data contain little information then the individual's prior receives most of the weight in the posterior distribution, and there is no reason to believe that the individuals priors will, even on average, be correct.

Section 3: Summary and Conclusions

The assumption of rational expectations has replaced *ad hoc* assumptions for expectations in most economic models. Any asumption about expectations places a restriction on the way individual subjective beliefs are represented. In rational expectations models, expectations are endogenous variables. The standard equilibrium conditions are extended to expectations so that in equilibrium the average individual's subjective belief about the likelihood of an event equals the objective probability of that event. The rational expectations equilibrium condition is much more stringent than the standard zero excess demand condition since expectations and probabilities are inherently unobservable. To make rational expectations operational, individuals must be able to infer the correct probabilities from observable data. This paper illustrates that at a theoretical level operational rational expectations equilibria are stationary-state equilibria.

While stationary-state rational expectations models are valuable tools for analyzing the properties of an economy in a stochastic equilibrium, or for comparing the properties of different hypothetical economies – say with different stochastic policy regimes – they cannot be used for normative analysis of discretionary policy regime changes. If there are alternative policy rules which can be selected by a policy maker – as opposed to policy rules which are drawn randomly – then the economy is not in a stationary-state, and an operational rational expectations equilibrium is not well defined – this is the basic theoretical conundrum for rational expectations policy analysis. Currency reforms are often used to illustrate the neutrality of money with respect to a policy regime change and the real world applicability of rational expectations. Currency reforms, however, although they are discretionary, do not change the policy rule: they simply change the unit of account. Future realizations come from the same monetary policy process, and future and past data are compatible if an adjustment is made for the unit of account.

To draw analogies about the actual economy from the intuitively appealing rational expectations paradigm one must hope that the stationary-state model is a reasonable approximation to reality. The U.S. data, unfortunately, do not support the notion that actual business cycles can be viewed as essentially similar events. The empirical evidence against a single stochastic regime with a stable parametric distribution is robust with respect to alternative specifications, sample break points, and sample periods. Rational decisions appear to have been made under uncertainty in Knight's terminology. As a consequence the economy's response to similar exogenous events can be quite different since the response depends on the subjective expectations of the private sector which are not closely tied to an «objective» reality – the lag in monetary policy can be long and variable.

Wage and price controls and the stop and go fiscal and monetary sequence in the U.S. over the past fifteen years probably has increased uncertainty. If policy regime changes were the only source of nonstationarity the policy implications of this paper would be trivial. We believe, however, that other exogenous regime changes – such as OPEC – also have a significant effect on the economy. If this is true we need to replace the stringent restriction implied by rational expectations with a weaker restriction that yields a more robust model that can be used for normative policy analysis and that is compatible with the data.

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APPENDIX

1. The Data Set

The time series used in this paper consist of annual data collected from the following sources: the «*Economic Report of the President, transmited to the Congress, January 1980,*» the «*Historical Statistics of the United States,*» published by the U.S. Department of Commerce Bureau of the Census, the «*Long Term Economic Growth 1860-1970,*» published by the U.S. Department of Commerce Bureau of Economic Analysis, and the «Data Resources Incorporated» central data bank. Each series was taken from more than one source as shown below.

The GNP series in current dollars was taken from the Historical Statistics (series F1, 1889-1908), the DRI data bank (1909-1939) and the Economic Report of the President (1940-1979). The Wholesale Price Index was taken from the Historical Statistics (series F23, 1860-1970) and the DRI data bank (1913-1979, where the data for the overlapping period coincided). The Wholesale Price Index was used to deflate all nominal variables into real variables. We did not have data for the capital stock, and we used as a proxy the total real capital input index from the Long-Term Economic Growth (series A65, 1889-1970). For the period 1971-1979, the capital input index (CII) was constructed by regressing CII on total residential and non-residential capital stock. The data on the residential and non-residential capital stock were found in the DRI data banks (1925-1979). The interest rate used is the 4 to 6 months prime commercial paper rate, which was taken from the Historical Statistics (series X445, 1890-1970) and the Economic Report of the President (1940-1979), where the data for the overlapping period coincided no). The data on the wage rate are an index of the annual earnings of non-farm employees when they were employed. They were taken from the Historical Statistics (series D735, 1860-1900; series D724, 1900-1960) and the Economic Report of the President (1947-1979). Series D724 is the annual earnings of all employees so the data for the period 1901-1946 had to be adjusted by a multiplicative factor. (The adjustment coefficients were found by regressing the growth rate of series D724 on the growth rate of the series reported in the Economic Report of the President during the overlapping period of 1947-1960). Finally, the civilian unemployment rate data were taken from the Historical Statistics (series D86, 1890-1946) and the Economic Report of the President (1947-1979).

2. Tests of Parameter Stability

The basic test of parameter stability for each series is a likelihood ratio test. The ARIMA model that best described the whole sample period was fit in each sub-sample. The unrestricted likelihood function is the product of the likelihood functions of each sub-sample, and the restricted likelihood is the likelihood fuction of the original whole sample estimation. This likelihood ratio test is a test for all the parameters of the linear representation of the stochastic process that governs each series, that is the mean, γ, the autoregressive parameters, α, the moving average parameters, β, and the standard deviation of the error term, σ. We also present tests for the parameters α, β, and γ only, where σ is restricted to be the same in the two sub-samples. The complement to the above test, that is, a test for σ that restricts α, β, and γ to be equal across the sub-samples, is also tabulated. The test statistics are shown in the notes of table 1. All likelihood ratio tests were based on the conditional likelihood function except in the cases where the ARMA model of the detrended series was autoregressive of order one, because in such cases the unconditional likelihood was easy to calculate. Tables 1 through 6 present the results. We note an overwhelming rejection of the null hypotheses of stability in every series. Furthermore, in all series except for the real interest rate, the rejection is not only due to an unstable variance but is also due to unstable parameters α, β, and γ.

We performed an additional test on all detrended series together using a vector autoregressive model. We first performed a test to identify the lag length of the vector autoregressive model, and then performed the stability test by using dummy variables. Tables 7 and 8 present the test statistics and the results. Again, there is an overwhelming rejection of the null hypothesis of parameter stability.

TABLE I

ARIMA Results — All Series

Variable	ARIMA Model	Sample Partition		$H_0(\theta_i = \theta_j)$	Rejected at	$H_0(\theta_i = \theta_j / \sigma^2)$	Rejected at	$H_0(\sigma_i = \sigma_j / \{a(L), \gamma, \beta(L)\})$	Rejected at
ln(RGNP)	(1,1,0)	1889-1929,	1930-1979	$15.60 \sim \chi^2(3)$	.005	$15.37 \sim \chi^2(2)$	.001	$0.22 \sim \chi^2(1)$	.750
		1889-1941,	1942-1979	$8.00 \sim \chi^2(3)$	.050	$5.29 \sim \chi^2(2)$	.100	$2.70 \sim \chi^2(1)$	.100
RCAP	(1,1,1)	1889-1929,	1930-1979	$46.18 \sim \chi^2(4)$	.001	$9.95 \sim \chi^2(3)$	.025	$36.23 \sim \chi^2(1)$	.001
		1889-1941,	1942-1979	$14.63 \sim \chi^2(4)$	.010	$11.92 \sim \chi^2(3)$	.010	$2.71 \sim \chi^2(1)$	.100
RCP	(1,0,0)	1890-1929,	1930-1979	$16.80 \sim \chi^2(3)$	.001	$3.73 \sim \chi^2(2)$	.250	$13.07 \sim \chi^2(1)$	.001
		1890-1941,	1942-1979	$17.58 \sim \chi^2(3)$	.001	$1.44 \sim \chi^2(2)$	.500	$16.14 \sim \chi^2(1)$	.001
ln(U)	(1,0,1)	1890-1929,	1930-1979	$32.83 \sim \chi^2(4)$	.001	$19.80 \sim \chi^2(3)$	.001	$13.03 \sim \chi^2(1)$	.001
		1890-1941,	1942-1979	$15.31 \sim \chi^2(4)$	.005	$1.77 \sim \chi^2(3)$	.750	$13.54 \sim \chi^2(1)$	.001
ln(RWAGE)	(1,1,1)	1862-1929,	1930-1979	$34.79 \sim \chi^2(4)$	.001	$8.38 \sim \chi^2(3)$	.050	$26.41 \sim \chi^2(1)$	.001
		1862-1941,	1942-1979	$25.81 \sim \chi^2(4)$	.001	$4.49 \sim \chi^2(3)$	.250	$21.32 \sim \chi^2(1)$	.001

Note: (a) RGNP is real Gross National Product, RCAP is an index of the real Capital Stock, RCP is real Commercial Paper Rate (4-6 months), U is Unemployment Rate, and RWAGE is real Wage Rate.

(b) α denotes the set of autoregressive parameters, β the set of moving-average parameters, γ the mean, σ the standard deviation of the error term, and θ the parameters α, β, γ, and σ.

(c) The Test Statistic for $H_0(\theta_i = \theta_j)$ is $(n - 2k)\ln \frac{SSR_i}{n_i - 2k} - (n_i \; k) - \ln \frac{SRS_i}{n_i - k} - (n_j - k)\ln \frac{SSR_j}{n_j - k}$, where n denotes the sample size, the subscripts i and j the relevant sub-samples, k the number of restrictions, and SSR thesum of squared residuals. This is a conditional likelihood ratio test statistic adjusted for degrees of freedom in order to be more conservative. The test statistic for $H_0(\theta_i = \theta_j \mid \sigma^2)$ is $(n-2k) \ln \frac{SSR}{SSR_i + SSR_j}$. The test statistic for $H_0(\theta_i = \theta_j \mid \{a(L), \beta(L), \gamma\})$, is $(n-2k)\ln \frac{SSR_i + SSR_j}{n - 2k} - (n_i - k) \ln \frac{SSR_i}{n_i - k} - (n_j - k) \ln \frac{SSR_j}{n_j - k}$. Note that the test statistic for $H_0(\theta_i = \theta_j)$ equals the sum of the statistics of the other two hypotheses. (d) In the cases of ln(RGNP) and RCP we used unconditional likelihood ratio test statistics because the correction factors were easy to compute. (e) In principle we can test for three regimes with the two break points in 1929-1930 and in 1941-1942. However, the period 1930-1941 is too small to give a good ARMA fit.

TABLE 2

Real Gross National Product (1889-1979)

ARMA (1,0) on $ln\ (RGNP_t) - ln\ (RGNP_{t-1})$

Sample	γ	α_1	Q-Statistic $\chi^2(23)$	Sum of Squared Residuals	$H_0\ (\theta_i = \theta_j)$	$H_0\ (\theta_i = \theta_j\ /\sigma^2)$
1890-1979	.035	.19 (.10)	21.2	.3030		
1890-1929	.039	—.26 (.15)	17.7	.1204	$-2\ln\lambda = 15.60 \sim \chi^2(3)$	$-2\ln\lambda = 15.38 \sim \chi^2(2)$
1930-1979	.031	.53 (.12)	22.3	.1319		
1890-1941	.036	—.02 (.14)	28.8	.1988	$-2\ln\lambda = 8.00 \sim \chi^2(3)$	$-2\ln\lambda = 5.29 \sim \chi^2(2)$
1942-1979	.034	.52 (.14)	20.2	.0852		

Note: (α) γ is the mean, α_1 is the AR(1) parameter, σ is the standard deviation of the error term, θ denotes the parameters γ, α_1, and σ, and λ is the unconditional likelihood ratio. (b) The numbers inside the parentheses are standard deviations. (c) The Test Statistic for $H_0\ (\sigma_i^2 = \sigma_j^2/(a_1, \gamma))$ is not tabulated since it is the difference of the two tabulated test statistics. (d) Since the unconditional likelihood ratio was used the Test Statistics are modified as follows:

For $H_0(\theta_i = \theta_j)$.

$$-2ln\lambda = (\eta - 2\kappa)\ ln\ \frac{SSR}{n-2k} - ln(1-a_1^2) - (\eta_i - k)\ ln\ \frac{SSR_i}{n_i - k} + ln(1 - a_{1i}^2) - (n_j - k) ln\ \frac{SSR_j}{n_j - k} + ln\ (1 - a_{1j}^2).$$

The modification for $H_0\ (\theta_i = \theta_j\ /\sigma^2)$ is similar, i.e.

$$-2ln\lambda = (n-2k) ln\ \frac{SSR}{SSR_i + SSR_j} - ln(1 - a_1^2) + ln(1 - a_{1i}^2) + ln(1 - a_{1j}^2).$$

TABLE 3

Real Capital Stock (1889-1979)

			ARMA (1,1) on $RCAP_t - RCAP_{t-1}$				
Sample	γ	α_1	β_1	Q-Statistic $\chi^2(22)$	Sum of Squared Residuals	H_0 $(\theta_i = \theta_j)$	H_0 $(\theta_i = \theta_j / \sigma^2)$
1890-1979	1.93	.83 (.07)	—.29 (.11)	28.0	65.102		
1890-1929	1.08	.00 (1.00)	—.16 (.99)	19.3	5.162	$-2\ln\lambda = 46.18 \sim \chi^2(4)$	$-2\ln\lambda = 9.95 \sim \chi^2(3)$
1930-1979	2.62	.76 (.10)	—.48 (.14)	35.0	52.785		
1890-1941	.79	.48 (.19)	—.28 (.21)	47.0	25.785	$-2\ln\lambda = 14.63 \sim \chi^2(4)$	$-2\ln\lambda = 11.92 \sim \chi^2(3)$
1942-1979	3.49	.63 (.14)	—.56 (.16)	38.8	30.703		

Note: (a) λ is the conditional likelihood ratio.
(b) The numbers in the parentheses are standard errors.

TABLE 4

Real Interest Rate (1890-1979)

			ARMA (1,0) on RCP_t			
Sample	γ	α_1	Q-Statistic $\chi^2(23)$	Sum of Squared Residuals	$H_0\ (\theta_i = \theta_j\ )$	$H_0\ (\theta_i = \theta_j\ /\sigma^2)$
1890-1979	1.55	.38 (.10)	22.8	5994		
1890-1929	3.58	.22 (.15)	28.6	4044	$-2\ln\lambda = 16.80 \sim \chi^2(3)$	$-2\ln\lambda = 3.73 \sim \chi^2(2)$
1930-1979	—0.07	.57 (.12)	35.0	1676		
1890-1941	3.14	.29 (.13)	22.7	4932	$-2\ln\lambda = 17.58 \sim \chi^2(3)$	$-2\ln\lambda = 1.44 \sim \chi^2(2)$
1942-1979	—0.62	.54 (.14)	27.7	943		

Note: (a) λ is the unconditional likelihood ratio. The correction factors on the statistics are similar to those for RGNP.
(b) The numbers in the parentheses are standard errors.

TABLE 5

Unemployment Rate (1890-1979)

	ARMA (1,1) on $l\eta U_t$						
Sample	γ	a_1	β_1	Q-Statistic χ	Sum of Squared Residuals	H_0 ($\theta_i = \theta_j$)	H_0 ($\theta_i = \theta_j/\sigma^2$)
1890-1979	1.73	.53	—.55	21.2	15.052		
		(.11)	(.11)				
1890-1929	1.62	.03	—.95	25.0	8.128	—2lηλ = 32.83 $\sim \chi^2(4)$	—2lηλ = 19.80 $\sim \chi^2(3)$
		(.18)	(.14)				
1930-1979	1.82	.80	—.46	25.3	3.763		
		(.08)	(.14)				
1890-1941	1.89	.47	—.59	20.2	12.177	—2lηλ = 15.31 $\sim \chi^2(3)$	—2lηλ = 1.77 $\sim \chi^2(3)$
		(.15)	(.14)				
1942-1979	1.50	.50	—.47	38.2	2.562		
		(.18)	(.19)				

Note: (a) λ is the conditional likelihood ratio.
(b) The numbers in the parentheses are standard errors.

TABLE 6

Real Wage Rate (1862 - 1979)

ARMA (1,1) on $\ln(RWAGE_t) - \ln(RWAGE_{t-1})$

Sample	γ	α_1	β_1	Q-Statistic $\chi^2(22)$	Sum of Squared Residuals	H_0 $(\theta_i = \theta_j)$	$H_0(\theta_i = \theta_j / \sigma^2)$
1863-1979	.016	—.57 (.18)	—.80 (.14)	30.5	.4446		
1863-1929	.017	.14 (1.66)	.07 (1.68)	32.7	.3545	$-2\ln\lambda = 34.79 \sim \chi^2(4)$	$-2\ln\lambda = 8.38 \sim \chi^2(3)$
1930-1979	.015	.20 (.16)	—.81 (.10)	22.6	.0577		
1863-1941	.017	.07 (.90)	—.06 (.90)	38.7	.3872	$-2\ln\lambda = 25.81 \sim \chi^2(4)$	$-2\ln\lambda = 4.49 \sim \chi^2(3)$
1942-1979	.016	.14 (.18)	—.88 (.11)	19.0	.0398		

Note : (a) λ is the conditional likelihood ratio.
(b) The numbers in the parentheses are standard errors.

TABLE 7

Test of Lag Lenght in Vector Autoregression (1892-1979)

Equation	Test for Second Order Lag
$y_t \equiv \ln(RGNP_t) - \ln(RGNP_{t-1})$	$F(5,77) = 1.356$
$k_t \equiv RCAP_t - RCAP_{t-1}$	$= 0.953$
$r_t \equiv RCP_t$	$= 1.758$
$v_t \equiv \ln U_t$	$= 1.143$
$w_t \equiv \ln(RWAGE_t) - \ln(RWAGE_{t-1})$	$= 1.687$
All Equations	$\chi^2 = 27.270$, reject at .500

Note: (a) Each variable as defined above is regressed on: constant, y_{t-1}, k_{t-1}, r_{t-1}, u_{t-1}, v_{t-1}, y_{t-2}, k_{t-2}, r_{t-2}, v_{t-2}, and w_{t-2}. (b) The F-tests are, of course, not distributed as F here because of the presence of lagged dependent variables. (c) The χ^2 stastatistic is $(\eta - \kappa) \cdot (\ln|S_R| - \ln|S_u|)$, where $\eta = 88$ (the sample size), $\kappa = 11$ (the number of restrictions), S_R is the matrix of cross products of residuals when the model is restricted, and S_u is the same matrix for the unrestricted model.

TABLE 8

Test of Parameter Stability in the Vector AR(1) Model (1891-1979)
Sample Partition: 1891-1929, 1930-1979

Equation	Test for Stability	
y_t	$F(6,77)$	$= 2.070$
k_t		$= 18.009$
r_t		$= 2.133$
v_t		$= 1.448$
w_t		$= 2.087$
All Equations	$\chi^2(30)$	$= 71.30$, reject at .001
All Equations, except that of k_t	$\chi^2(24)$	$= 48.12$, reject at .005

Note : (a) All variables are defined on Table 7. (b) The F-tests are, of cource, not distributed as F here because of the presence of lagged dependent variables. (c) The χ^2 statistic is similar to the one on Table 7 with $\eta = 89$ and $\kappa = 12$.